Name: ____________________

Year 11 2023 Mathematics Advanced Assessment Task 1

Investigative Assignment with Validation Task		
Task number: 1	**Weighting:** 30%	**Due Date:** Tuesday 7/3/2023

Outcomes assessed:

MA 11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems
MA 11-9 provides reasoning to support conclusions which are appropriate to the context

Nature and description of the task:

As a result of completing this Investigative Assignment, students should be familiar with positive, negative and fractional indices. They should be able to expand and factorise expressions in a variety of contexts, demonstrate the capacity to simplify algebraic fractions, and substitute in given values for pronumerals. Students should also be able to work with surds and be able to solve linear equations.

On the 7th March, 2023 you will receive a selection of questions similar to the Preparation Activity below to complete in a 1 hour in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive in the in-class Validation task.
NOTE: You will not have to hand in the answers to the questions in this Preparation Activity AND you will not have access to the Preparation Activity during the Validation Task.

Non-Completion of Task:

If you know you are going to be away on the day of the Validation Task and are unable to complete it on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Part 1 Preparation Activity – investigate/attempt each of the following questions in preparation for the in-class Validation Task.

General Number Review:

1 Classify each of these real numbers as rational or irrational. Express those that are rational in the form $\frac{a}{b}$, where a and b are integers.

a 7 **b** $-2\frac{1}{4}$ **c** $\sqrt{9}$ **d** $\sqrt{10}$

e $\sqrt[3]{15}$ **f** $\sqrt[4]{16}$ **g** -0.16 **h** π

2 Use a calculator to evaluate the following, giving the answers in scientific notation using three significant figures.

a $(2.31)^{-7}$ **b** $(5.04)^{-4}$ **c** $(2.83 \times 10^2)^{-3}$

d $5.1 \div (8 \times 10^2)$ **e** $9.3 \times 10^{-2} \times 8.6 \times 10^8$ **f** $(3.27 \times 10^4) \div (9 \times 10^{-5})$

g $\sqrt{3.23 \times 10^{-6}}$ **h** $\pi(3.3 \times 10^7)^2$ **i** $\sqrt[3]{5.73 \times 10^{-4}}$

3 Evaluate, correct to three significant figures:

a $\dfrac{7.93}{8.22 - 3.48}$ **b** $-4.9 \times (-5.8 - 8.5)$ **c** $\sqrt[4]{1.6 \times 2.6}$

d $\dfrac{13^5}{11^6 + 17^4}$ **e** $\dfrac{\frac{4}{9} - \frac{2}{7}}{\frac{5}{8} - \frac{3}{10}}$ **f** $\sqrt{2.4^{-1.6}}$

g $\sqrt{\dfrac{1.347}{2.518 - 1.679}}$ **h** $\dfrac{2.7 \times 10^{-2}}{1.7 \times 10^{-5}}$ **i** $\dfrac{\sqrt{\frac{1}{2}} + \sqrt[3]{\frac{1}{3}}}{\sqrt[4]{\frac{1}{4}} + \sqrt[5]{\frac{1}{5}}}$

4 Convert these recurring decimals to fractions.

a $0.\dot{8}$ **b** $0.1\dot{8}$ **c** $0.\dot{1}\dot{8}$ **d** $0.41\dot{6}$

Index Laws:

1 Write each expression as an integer or fraction.

a 5^3 **b** 2^8 **c** 10^9 **d** 17^{-1}

e 9^{-2} **f** 2^{-3} **g** 3^{-4} **h** 27^0

i $\left(\frac{2}{3}\right)^3$ **j** $\left(\frac{7}{12}\right)^{-1}$ **k** $\left(\frac{5}{6}\right)^{-2}$ **l** $36^{\frac{1}{2}}$

m $27^{\frac{1}{3}}$ **n** $8^{\frac{2}{3}}$ **o** $9^{\frac{5}{2}}$ **p** $\left(\frac{4}{49}\right)^{\frac{1}{2}}$

q $\left(\frac{14}{59}\right)^0$ **r** $\left(\frac{9}{25}\right)^{-\frac{1}{2}}$ **s** $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **t** $\left(\frac{9}{100}\right)^{-\frac{3}{2}}$

2 Rewrite these expressions, using fractional and negative indices.

a $\frac{1}{x}$ **b** $\frac{7}{x^2}$ **c** $-\frac{5}{\sqrt{x}}$ **d** $-\frac{1}{2x}$

e $-\frac{4}{\sqrt[3]{x^2}}$ **f** $x\sqrt{x}$ **g** $\frac{5}{x\sqrt{x}}$ **h** $8x^2\sqrt{x}$

3 Simplify:

a $(x^4)^5$ **b** $\left(\frac{3}{a^3}\right)^4$ **c** $(25x^6)^{\frac{1}{2}}$ **d** $\left(\frac{8r^3}{t^6}\right)^{\frac{1}{3}}$

4 Simplify each expression, leaving the answer in index form.

a $x^2y \times y^2x$ **b** $15xyz \times 4y^2z^4$ **c** $3x^{-2}y \times 6xy^{-3}$

d $4a^{-2}bc \times a^5b^2c^{-2}$ **e** $x^3y \div xy^3$ **f** $14x^{-2}y^{-1} \div 7xy^{-2}$

g $m^{\frac{1}{2}}n^{-\frac{1}{2}} \times m^{1\frac{1}{2}}n^{-\frac{1}{2}}$ **h** $(2st)^3 \times (3s^3)^2$ **i** $(4x^2y^{-2})^3 \div (2xy^{-1})^3$

5 Solve each equation for x.

a $3^x = 81$ **b** $5^x = 25$ **c** $7^x = \frac{1}{7}$ **d** $2^x = \frac{1}{32}$

e $\left(\frac{1}{3}\right)^x = \frac{1}{9}$ **f** $\left(\frac{2}{3}\right)^x = \frac{8}{27}$ **g** $25^x = 5$ **h** $8^x = 2$

<u>Simplifying Algebraic Terms:</u>

1 Simplify:

a $-8y + 2y$ **b** $-8y - 2y$ **c** $-8y \times 2y$

2 Simplify:

a $-2a^2 - a^2$ **b** $-2a^2 - (-a^2)$ **c** $-2a^2 \times (-a^2)$

3 Simplify:

a $3t - 1 - t$ **b** $-6p + 3q + 10p$

c $7x - 4y - 6x + 2y$ **d** $2a^2 + 8a - 13 + 3a^2$

4 Simplify:

a $-6k^6 \times 3k^3$ **b** $-6k^6 \div 3k^3$ **c** $(-6k^6)^2$

5 Expand and simplify:

a $4(x + 3) + 5(2x - 3)$ **b** $8(a - 2b) - 6(2a - 3b)$ **c** $-(a - b) - (a + b)$

d $-4x^2(x + 3) - 2x^2(x - 1)$ **e** $(n + 7)(2n - 3)$ **f** $(r + 3)^2$

g $(y - 5)(y + 5)$ **h** $(3x - 5)(2x - 3)$ **i** $(t - 8)^2$

j $(2c + 7)(2c - 7)$ **k** $(4p + 1)^2$ **l** $(3u - 2)^2$

6 Factor:

a $18a + 36$ **b** $20b - 36$ **c** $9c^2 + 36c$
d $d^2 - 36$ **e** $e^2 + 13e + 36$ **f** $f^2 - 12f + 36$
g $36 - 25g^2$ **h** $h^2 - 9h - 36$ **i** $i^2 + 5i - 36$
j $2j^2 + 11j + 12$ **k** $3k^2 - 7k - 6$ **l** $5l^2 - 14l + 8$
m $4m^2 + 4m - 15$ **n** $mn + m + pn + p$ **o** $p^3 + 9p^2 + 4p + 36$
p $qt - rt - 5q + 5r$ **q** $u^2w + vw - u^2x - vx$ **r** $x^2 - y^2 + 2x - 2y$

7 Simplify:

a $\frac{x}{2} + \frac{x}{4}$ **b** $\frac{x}{2} - \frac{x}{4}$ **c** $\frac{x}{2} \times \frac{x}{4}$ **d** $\frac{x}{2} \div \frac{x}{4}$

e $\frac{3a}{2b} + \frac{2a}{3b}$ **f** $\frac{3a}{2b} - \frac{2a}{3b}$ **g** $\frac{3a}{2b} \times \frac{2a}{3b}$ **h** $\frac{3a}{2b} \div \frac{2a}{3b}$

i $\frac{x}{y} + \frac{y}{x}$ **j** $\frac{x}{y} - \frac{y}{x}$ **k** $\frac{x}{y} \times \frac{y}{x}$ **l** $\frac{x}{y} \div \frac{y}{x}$

8 Simplify:

a $\frac{x + 4}{5} + \frac{x - 5}{3}$ **b** $\frac{5}{x + 4} + \frac{3}{x - 5}$ **c** $\frac{x + 1}{2} - \frac{x - 4}{5}$

d $\frac{2}{x + 1} - \frac{5}{x - 4}$ **e** $\frac{x}{2} - \frac{x + 3}{4}$ **f** $\frac{2}{x} - \frac{4}{x + 3}$

9 Factor each expression where possible, then simplify it.

a $\frac{6a + 3b}{10a + 5b}$ **b** $\frac{2x - 2y}{x^2 - y^2}$ **c** $\frac{x^2 + 2x - 3}{x^2 - 5x + 4}$

d $\frac{2x^2 + 3x + 1}{2x^3 + x^2 + 2x + 1}$ **e** $\frac{a + b}{a^2 + 2ab + b^2}$ **f** $\frac{3x^2 - 19x - 14}{9x^2 - 4}$

10 Simplify.

a $\frac{12x}{x^2 + 2x - 3} \times \frac{x^2 - 1}{6x + 6}$ **b** $\frac{4x^2 - 9}{2x^2 + x - 6} \div \frac{8x + 12}{x^2 - 2x - 8}$

Surds:

1 Simplify:

a $\sqrt{24}$ **b** $\sqrt{45}$ **c** $\sqrt{50}$

d $\sqrt{500}$ **e** $3\sqrt{18}$ **f** $2\sqrt{40}$

2 Simplify:

a $\sqrt{5}+\sqrt{5}$ **b** $\sqrt{5}\times\sqrt{5}$ **c** $\left(2\sqrt{7}\right)^2$

d $2\sqrt{5}+\sqrt{7}-3\sqrt{5}$ **e** $\sqrt{35}\div\sqrt{5}$ **f** $6\sqrt{55}\div 2\sqrt{11}$

g $\sqrt{8}\times\sqrt{2}$ **h** $\sqrt{10}\times\sqrt{2}$ **i** $2\sqrt{6}\times 4\sqrt{15}$

3 Simplify:

a $\sqrt{27}-\sqrt{12}$ **b** $\sqrt{18}+\sqrt{32}$

c $3\sqrt{2}+3\sqrt{8}-\sqrt{50}$ **d** $\sqrt{54}-\sqrt{20}+\sqrt{150}-\sqrt{80}$

4 Expand:

a $\sqrt{7}\left(3-\sqrt{7}\right)$ **b** $\sqrt{5}\left(2\sqrt{6}+3\sqrt{2}\right)$

c $\sqrt{15}\left(\sqrt{3}-5\right)$ **d** $\sqrt{3}\left(\sqrt{6}+2\sqrt{3}\right)$

5 Expand and simplify:

a $\left(\sqrt{5}+2\right)\left(3-\sqrt{5}\right)$ **b** $\left(2\sqrt{3}-1\right)\left(3\sqrt{3}+5\right)$ **c** $\left(\sqrt{7}-3\right)\left(2\sqrt{5}+4\right)$

d $\left(\sqrt{10}-3\right)\left(\sqrt{10}+3\right)$ **e** $\left(2\sqrt{6}+\sqrt{11}\right)\left(2\sqrt{6}-\sqrt{11}\right)$ **f** $\left(\sqrt{7}-2\right)^2$

g $\left(\sqrt{5}+\sqrt{2}\right)^2$ **h** $\left(4-3\sqrt{2}\right)^2$

6 Write with a rational denominator:

a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{3}{\sqrt{2}}$ **c** $\dfrac{\sqrt{3}}{\sqrt{11}}$

d $\dfrac{1}{5\sqrt{3}}$ **e** $\dfrac{5}{2\sqrt{7}}$ **f** $\dfrac{\sqrt{2}}{3\sqrt{10}}$

7 Write with a rational denominator:

a $\dfrac{1}{\sqrt{5}+\sqrt{2}}$ **b** $\dfrac{1}{3-\sqrt{7}}$ **c** $\dfrac{1}{2\sqrt{6}-\sqrt{3}}$

d $\dfrac{\sqrt{3}}{\sqrt{3}+1}$ **e** $\dfrac{3}{\sqrt{11}+\sqrt{5}}$ **f** $\dfrac{3\sqrt{7}}{2\sqrt{5}-\sqrt{7}}$

8 Rationalise the denominator of each fraction.

a $\dfrac{\sqrt{7} - \sqrt{2}}{\sqrt{7} + \sqrt{2}}$

b $\dfrac{3\sqrt{3} + 5}{3\sqrt{3} - 5}$

9 Find the value of x if $\sqrt{18} + \sqrt{8} = \sqrt{x}$.

10 Simplify $\dfrac{3}{\sqrt{5} - 2} + \dfrac{2}{\sqrt{5} + 2}$ by forming the lowest common denominator.

11 Find the values of p and q such that $\dfrac{\sqrt{5}}{\sqrt{5} - 2} = p + q\sqrt{5}$.

12 Show that $\dfrac{2}{6 - 3\sqrt{3}} - \dfrac{1}{2\sqrt{3} + 3}$ is rational by first rationalising each denominator.

<u>Linear Equations:</u>

1 Solve each linear equation.

a $3x + 5 = 17$

b $3(x + 5) = 17$

c $\dfrac{x + 5}{3} = 17$

d $\dfrac{x}{3} + 5 = 17$

e $7a - 4 = 2a + 11$

f $7(a - 4) = 2(a + 11)$

g $\dfrac{a - 4}{7} = \dfrac{a + 11}{2}$

h $\dfrac{a}{7} - 4 = \dfrac{a}{2} + 11$

End of Part 1 Preparation Activity

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

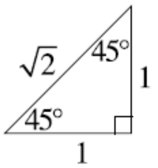
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

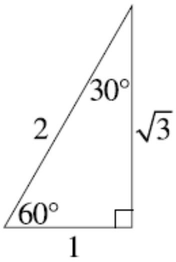


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

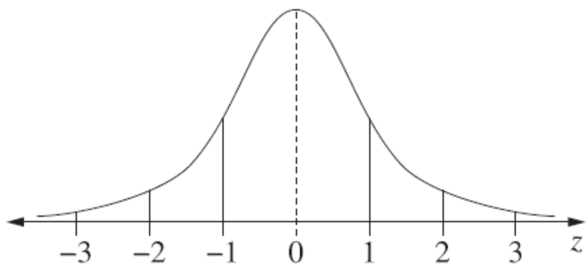
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$