Name: ____________________

Year 12 2023/2024 Mathematics Advanced Assessment Task 1

Investigative Assignment with Validation Task

Task number: 1	**Weighting:** 20%	**Due Date:** Thursday 30/11/23

Outcomes assessed:

MA 11-4	uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities
MA 11-6	manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems
MA 11-7	uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions
MA 12-1	uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts
MA 12-3	applies calculus techniques to model and solve problems
MA 12-4	applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems
MA 12-5	applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs
MA12-10	constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

Nature and description of the task:

As a result of completing this Investigative Assignment, students should be familiar with the logarithmic and index laws, functions involving logarithms and exponentials including the derivative of the natural exponential function. They will also be able to understand random variables and their various definitions. Recognise discrete probability distributions and their properties to solve practical properties, and find expected values, variance and standard deviations of probability distributions. They will be able to solve trigonometric equations, use the trigonometric identities and graph trigonometric functions. They will be able to use the trig identities in the solution of problems. Finally, they will be able to recognise and apply concepts involving sequences and series, in particular arithmetic and geometric progressions, including, *nth* term, sum to *n* terms and sum to infinity.

On the 30th November, 2023 you will receive a selection of similar questions to the Preparation Activity below to complete in a 1 hour in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive in the in-class Validation task. NOTE: You will not have to hand in the answers to the questions in this Preparation Activity AND you will not have access to the Preparation Activity during the Validation Task.

Non-Completion of Task:

If you know you are going to be away on the day of the Validation Task and are unable to complete it on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Part 1 Preparation Activity – investigate/attempt each of the following questions in preparation for the in-class Validation Task.

Exponential and Logarithmic Functions:

1 Evaluate e^5 to 2 decimal places.

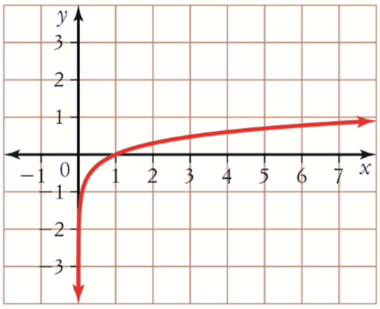
2 Differentiate $y = (e^x + 7x)^3$.

3 Find the gradient of the tangent to the graph of $y = e^x + 2$ at $x = -1$.

4 Evaluate $7^{\log_3 9}$

5 Solve the equation $3 \ln 2 + \ln 5 - \ln 10 = \ln x$.

6 Which function below has this graph?



A $y = \log x$ **B** $y = e^x$

C $y = ba^x$ **D** $y = \log |x + 1|$

7 Which statement is true if $5^x = 35$?

A $x = \log_{10} 35$ **B** $x = \dfrac{\log_{10} 35}{\log_{10} 5}$

C $x = \dfrac{\log_{10} 5}{\log_{10} 35}$ **D** $x = \dfrac{\log_{10} 35}{5}$

8 Evaluate $\log_7 37$.

9 If $\log_e 5 = 1.6$ and $\log_e 9 = 2.2$, then which is the correct expression for $\log_e 45$?

A 1.6×2.2 **B** $1.6 \div 2.2$

C $1.6 - 2.2$ **D** $1.6 + 2.2$

10 Solve the equation $21 = 8e^{0.024t}$.

11 Graph each function.

a $y = 3^x + 2$

b $y = 3^{x-1}$

c $y = \log(x + 1)$

d $y = \log x - 2$

12 **a** Find the exact value of the gradient of the normal to the graph of $y = e^x + 5x^2$ at $x = 1$.

b Hence find the equation of the normal to the graph at $x = 1$.

13 Simplify each expression.

a $2\log_{10} 5 + \log_{10} 4 - \log_{10} 2$

b $\log_3 9xy^2 - \log_3 27xy$

c $\log_4 16x^4 - \log_4 (2x)^2$

d $\ln\left(\dfrac{e^x x^2}{y}\right) + \ln e^x y$

14 Solve each equation.

a $\ln(x + 1) = 1.72$

b $e^{2x} = 6.27$

c $\log_5 x = 3$

d $15 = 12e^{x-2}$

e $11^{5x-2} = 5$

f $\log_x 5 = \dfrac{1}{5}$

15 The number of bacteria in a sample of water from an infested lake is increasing by 5% per hour. Initially, there were 1500 bacteria when testing started.

a Write an equation that represents the number of bacteria after t hours.

b How many bacteria are present after 2 days?

c A pesticide was introduced 10 hours after the initial count. The pesticide had a positive result in controlling the growth of bacteria, and reduced it by 7% per hour. After how many hours will the number of bacteria be reduced to 500?

16 Evaluate each expression correct to 3 significant figures, where possible.

a $\log_{10} 50 - \log_{10} 5$

b $e^{\ln 5} \div e^{\ln 6}$

c $e^2 + e^{-3}$

d $\dfrac{2e^3}{\log_{10} 4}$

17 State the domain, range and asymptotes of each function.

a $y = 2 \log_e x$

b $y = 2 \times 10^x$

18 a Write $\log_3 729 = 6$ in index form.

b Write $3^{-3} = \dfrac{1}{27}$ in logarithm form.

19 The population of a Pacific island is given by $P = P_0e^{kt}$, where P_0 is the initial population, t is the time in years after the year 2004 and k is a constant. In 2004, the population was 10 200. In 2009 (5 years later), the population increased to 17 250.

a What is the value of P_0?

b What is the value of k?

c What will be the population in 2020?

d How many years will it take for the population to reach 100 000?

e If the area of the island is 1525 km^2, how many years will it take for the island to be overpopulated at a population density of 100 persons per 1000 m^2?

Trigonometric Functions:

1 Convert $\frac{3\pi}{7}$ to degrees.

2 Find the exact value of $\sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)$.

3 Find the exact value of cos (225°).

4 Find the value of t if $\tan(2t + 15) = \cot(6t - 5)$.

5 Compared to $y = \cos x$, the function $y = 3\cos 2x$ has:

A the same amplitude and period

B a larger amplitude and a shorter period

C a smaller amplitude and a shorter period

D a smaller amplitude and a larger period

6 A circle has a radius of 25 cm. An arc on the circle measuring 37 cm subtends an angle θ at the centre of the circle. Calculate θ in radians.

7 Find the exact value of sin (–300°).

8 Given that $\sin\theta = \frac{12}{13}$, find the exact value of cos θ.

9 Which statement is true?

A $\sin\left(x + \frac{\pi}{2}\right) = -\cos x$

B $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

C $\sin\left(x - \frac{\pi}{2}\right) = \cos x$

D $\cos\left(x + \frac{\pi}{2}\right) = \sin 2x$

10 Solve $2\cos x = \sqrt{3}$ for $0 \le x \le 2\pi$.

11 If $\sin\theta = -\frac{3}{7}$, $\cos\theta < 0$ and $\tan\theta > 0$, then find the value of:

a cos θ

b tan θ

c θ to the nearest degree

12 Simplify each expression.

a $\dfrac{\cot x \sec x}{\tan x \cos x}$

b $\dfrac{\tan^2 x}{\sec^2 x}$

13 **a** Prove that $(\sin x - \cos x)^2 = \sec^2 x - \tan^2 x - 2 \sin x \cos x$.

b Using what you proved in part **a** or otherwise, prove that $\sec^2 x - \tan^2 x - \operatorname{cosec}^2 x + \cot^2 x = 0$.

14 Prove that $\dfrac{1 + \sin x}{1 - \sin x} = (\sec x + \tan x)^2$.

15 **a** In which quadrant does the angle $\dfrac{16\pi}{3}$ lie?

b Hence find the exact values of $\sin\left(\dfrac{16\pi}{3}\right)$ and $\cos\left(\dfrac{16\pi}{3}\right)$.

16 Solve $3 \tan x - 1 = 0$ in the domain $[0, \pi]$, correct to 3 decimal places.

17 Sketch each trigonometric function in the domain $[0, 2\pi]$.

a $f(x) = 3\cos\left(x - \dfrac{\pi}{2}\right)$

b $g(x) = \tan x - 1$

c $y = \cos(2x)$

d $f(x) = 1 + \sin 3x$

18 Find the domain and range of each function.

a $y = -2 \cos x$

b $y = 5 \sin(-3x)$

19 Find the amplitude and period of $y = -2 \sin 3x$.

20 Solve the equation $\sin^2 \theta - 4 \sin \theta + 3 = 0$ for the domain $[0, 2\pi]$.

21 Find the period, amplitude, centre and phase of $y = \sin\left(2x - \dfrac{\pi}{4}\right) - 3$.

Discrete Probability Distributions:

1 What is the set of possible values for the number of boys in a 4-children family?

A {1, 2, 3, 4} **B** {0, 1, 2}

C {0, 1, 2, 3} **D** {0, 1, 2, 3, 4}

2 The probability distribution for the number of pets owned by a group of Year 11 students is shown.

x	0	1	2	3	4	5
$P(X = x)$	0.11	0.25	0.27	0.18	0.15	0.04

Find the probability that a student chosen at random from this group owns 2 or more pets.

3 Find the value of A in this probability distribution table.

x	0	1	2	3	4
$P(X = x)$	0.1	0.12	0.23	A	0.45

4 Which one of these random variables is not discrete?

A The blood-alcohol content (BAC) level of a driver

B A reviewer's rating of a restaurant, from 1 to 4 stars

C The number of phones owned by the members of a family

D The sum of the 2 numbers rolled on a pair of dice

5 Find the standard deviation of discrete random variables with values of 1, 2 or 3, each with a probability of $\frac{1}{3}$.

6 Which one of these functions is a probability distribution?

A

x	0	1	2	3
$P(x)$	0.27	0.23	0.28	0.21

B $\left(0,\frac{2}{7}\right),\left(1,\frac{1}{7}\right),\left(2,\frac{4}{7}\right),\left(3,\frac{1}{7}\right),(4,0)$

C $P(x) = \frac{x}{6}$ for $x = 0, 1, 2, 3$

D $P(x) = \frac{2x+1}{5}$ for $x = 0, 1, 2$

7 Find the variance of this probability function:

$P(X) = \dfrac{x+2}{14}$, for $x = 0, 1, 2, 3$

8 Find the mean and standard deviation of this probability function.

$$P(x) = \begin{cases} \dfrac{2x}{5}, & \text{for } x = 1 \\ \dfrac{3x-2}{25}, & \text{for } x = 4 \\ \dfrac{x-5}{10}, & \text{for } x = 7 \end{cases}$$

9 State whether each function is a probability distribution, and state the reason.

a

x	−2	−1	0	1	2
$P(x)$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{3}{10}$	$\frac{7}{20}$	$\frac{1}{5}$

b $P(x) = \begin{cases} \dfrac{2x-1}{5}, & \text{for } x = 1 \\ \dfrac{3x-4}{25}, & \text{for } x = 3 \\ \dfrac{x-1}{10}, & \text{for } x = 7 \end{cases}$

c $\left(0, \frac{2}{9}\right), \left(1, \frac{1}{15}\right), \left(2, \frac{7}{12}\right), \left(3, \frac{1}{3}\right)$

10 Find the expected value of each distribution.

a

x	2	3	4	5
$P(x)$	0.1	0.3	0.4	0.2

b

x	7	9	11	13	15	17	19
$P(x)$	0.1	0.2	0.1	0.2	0.2	0.1	0.1

11 Find, correct to 2 decimal places, the variance and standard deviation of the random variable with values of x from 7 to 12, where $P(x) = \dfrac{1}{6}$.

12 The number of matches in a matchbox is not always 40 as labelled on the box.

A study of 100 boxes found the following distribution.

No. of matches	37	38	39	40	41	42	43
Frequency	2	18	20	40	15	4	1

a Draw up a probability distribution table.

b If a box is chosen at random, what is the probability that the box has

i at least 40 matches?

ii fewer than 39 matches?

13 For this probability function, find:

x	0	1	2	3	4	5	6
$P(x)$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{1}{4}$	$\frac{3}{20}$	$\frac{1}{10}$

a $P(X \geq 4)$

b $P(1 < X \leq 4)$

c Draw a graph of this function.

14 Ryan plays a game involving tossing 2 coins.

If he tosses 2 heads or 2 tails (HH or TT), he loses $4.

If he tosses 1 head, 1 tail (HT or TH), he can either win $2 or toss the coin that shows the tail again. If it results in a tail, he wins $8. But if it results in another head, he loses $10.

What is the expected value of the winning amount of this game if:

a Ryan does not choose to toss the coin again when he tosses HT or TH?

b Ryan always chooses to toss the coin again when he tosses HT or TH?

15 A bag of coloured marbles contains 7 red, 8 blue and 10 yellow marbles. 2 marbles are selected at random together in a handful. Let the discrete random variable X be the number of red marbles selected.

a Create a probability distribution table for X.

b Find the mean, variance and standard deviation of X.

c Let the discrete random variable Y be the number of yellow marbles selected. Create a probability distribution table for Y.

Sequences and Series: (NOTE >> Your teacher will let you know which Exercises in this Chapter will be included in the Validation Task)

1 What is the nth term of the sequence 7, 11, 15, …?

2 Which of the following is a geometric sequence?

A $ar, 2ar, 3ar, \ldots$ **C** $4r^2, 8r^2, 16r^2, \ldots$

B $r, 4r, 9r, 16r, \ldots$ **D** $a, a + r, a + r^2, \ldots$

3 What is the value of x in the geometric sequence $6, x, \frac{2}{27}, \cdots$?

4 What are the first 4 terms of a sequence which has 3rd term 18 and common ratio $-\frac{1}{3}$?

5 The first term of an arithmetic sequence is −8 and its 5th term is 16. What is the common difference?

6 What is the sum to infinity of $125 - 25 + 5 - 1 + \ldots$?

7 What is the sum of the first 20 terms of the sequence $\sqrt{3} + 3\sqrt{3} + 5\sqrt{3} + \cdots$?

8 What are the values of n for which the sequence given by $T_n = n^2 + 6n$ is negative?

A $n < -6$ **B** $0 < n < 6$

C $-6 < n < 0$ **D** $n > 6$

9 **a** The first 3 terms of a sequence is $x - 1, 3x + 2, 5x + 5, \ldots$ Show that the sequence is arithmetic.

b Find the sum of the first 50 terms of the sequence.

10 Which term of the sequence 2, 7, 12, … is equal to 197?

11 If $T_n = 2\pi n + 1$, then find the exact value of $T_5 + T_6 + \ldots + T_{19} + T_{20}$.

12 The first 3 terms of a geometric sequence are $x - 8, x, 3x - 16$. Each term in this particular sequence is a positive number.

a Show that the only possible value for x is 16.

b Find the common ratio.

c Show that the nth term of this geometric sequence is $T_n = 2^{n+2}$.

13 The 7th term of an arithmetic series is 14 and the 10th term is 23. What is the 30th term?

14 **a** Show that $\log_2 x + \log_2 x^2 + \log_2 x^3 + \ldots$ is an arithmetic series.

b What is the 25th term of this series?

c If $x = 4$, evaluate the 25th term.

d If $x = 4$, find the sum of the first 25 terms.

15 How many terms does it take for the sum $1 + 3 + 9 + \ldots$ to exceed 2000?

16 The 5th term of an arithmetic series is 10 and the sum of the first 10 terms is 70. What is the sum of 20 terms?

17 A bricklayer has 10 days to lay a brick wall, needing a total of 1100 bricks.

If he decides to start with k bricks the first day and then increase the number of bricks each day by k, find the value of k so he finishes building the entire wall in 10 days.

18 Prove that $T_n = S_n - S_{n-1}$.

End of Part 1 Preparation Activity

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\dfrac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

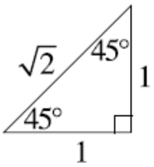
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

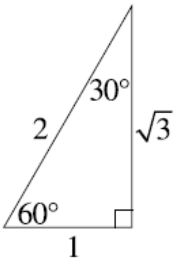
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

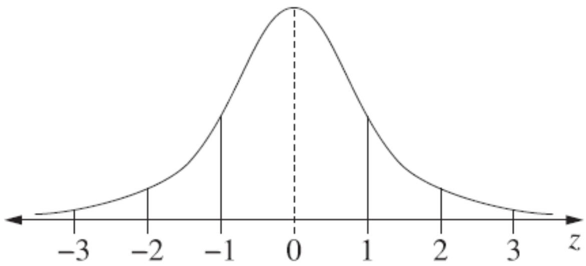
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2\,,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

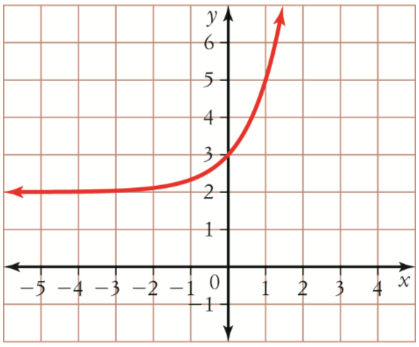
$$\ddot{x} = -n^{2}(x-c)$$

Exponential and Logarithmic Functions Answers:

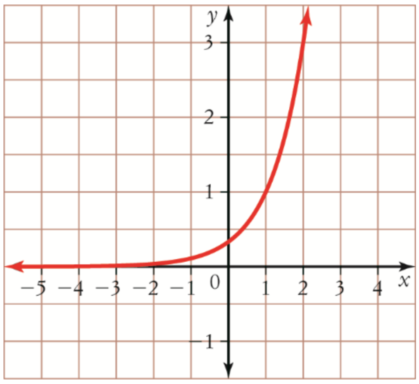
1 148.41 **2** $3(e^x + 7x)^2(e^x + 7)$ **3** $m = e^{-1}$ **4** 49 **5** $x = 4$

6 $y = \log x$ **7** $x = \dfrac{\log_{10}35}{\log_{10}5}$ **8** 1.856 **9** 1.6 + 2.2 **10** $t = 40.21$

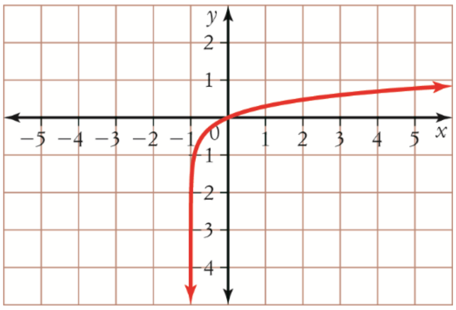
11 **a** $y = 3^x + 2$



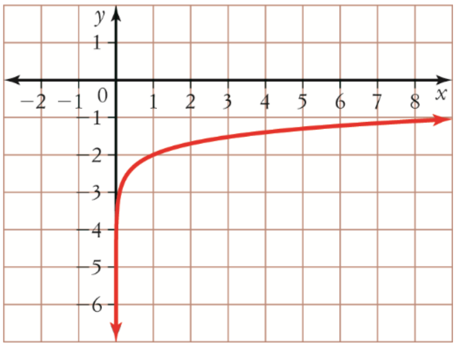
b $y = 3^{x-1}$



c $y = \log(x + 1)$



d $y = \log x - 2$



12 **a** $m = \dfrac{-1}{e + 10}$ **b** $(e + 10)y + x - e^2 - 15e - 49 = 0$

13 **a** $\log_{10}50$ **b** $\log_3 y - 1$ **c** $1 + \log_4 x^2$ **d** $3 \ln x + 3$

14 **a** 4.47 **b** 0.92 **c** 125 **d** –1.78 **e** 0.53 **f** 3125

15 **a** $N = 1500 \times 1.05^t$ **b** $N = 15\,602$ **c** 22 h

16 **a** 1 **b** 0.833 **c** 7.44 **d** 66.7

17 **a** Domain: $(0, \infty)$, range: $(-\infty, \infty)$ or $\boldsymbol{R}$: all real values; asymptote: $x = 0$ or the y-axis
b Domain: $(-\infty, \infty)$, range: $(0, \infty)$; asymptote: $y = 0$ or the x-axis

18 **a** $3^6 = 729$ **b** $\log_3\left(\dfrac{1}{27}\right) = -3$

19 **a** 10 200 **b** 0.105 **c** 54 729 **d** 21.7 or 22 years
e 25.8 or 26 years

Trigonometric Functions Answers:

1 77° **2** $\frac{1+\sqrt{2}}{2}$ **3** $-\frac{1}{\sqrt{2}}$ **4** 10 **5** B

6 1.480 **7** $\frac{1}{2}$ **8** $\frac{5}{13}$ **9** A **10** $x = \frac{\pi}{6}, \frac{11\pi}{6}$

11 **a** $-\frac{2\sqrt{10}}{7}$ **b** $\frac{3}{2\sqrt{10}}$ **c** $\theta = 205°$

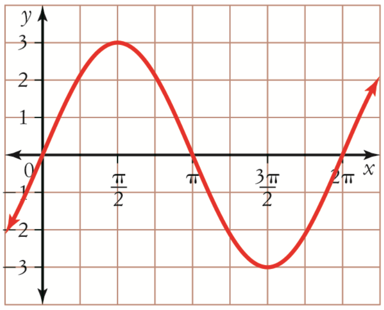
12 **a** $\operatorname{cosec}^2 x$ **b** $\sin^2 x$

13, 14 Proof

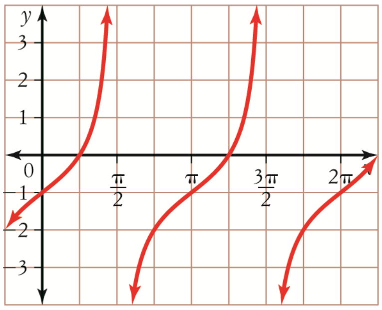
15 **a** 3rd quadrant **b** $-\frac{\sqrt{3}}{2}, \frac{1}{2}$

16 $x = 18.435, 198.435$

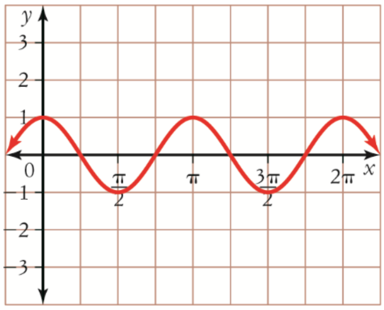
17 **a** $f(x) = 3\cos\left(x - \frac{\pi}{2}\right)$



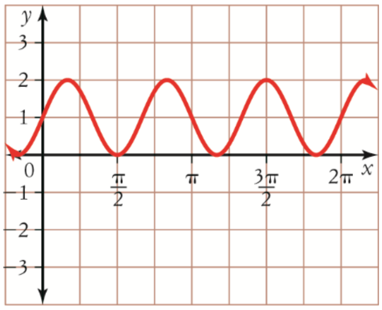
b $g(x) = \tan x - 1$



c $y = \cos(2x)$



d $f(x) = 1 + \sin 3x$



18 **a** Domain $(-\infty, \infty)$, range $[-1, 1]$ **b** Domain $(-\infty, \infty)$, range $[-5, 5]$

19 Amplitude 2, period $\frac{2\pi}{3}$

20 $\theta = \frac{\pi}{2}$

21 Period π, amplitude 1, centre -3, phase $-\frac{\pi}{8}$

Discrete Probability Distributions Answers:

1 D{0,1,2,3,4} **2** 0.64 **3** 0.1 **4** A **5** 0.82

6 C **7** 1.12 **8** mean=3.4, standard deviation= 2.24

9 **a** Yes, sum of $P(x)$ is 1 **b** Yes, sum of $P(x)$ is 1
c No, sum of $P(x)$ is not 1

10 **a** 3.7 **b** 12.8

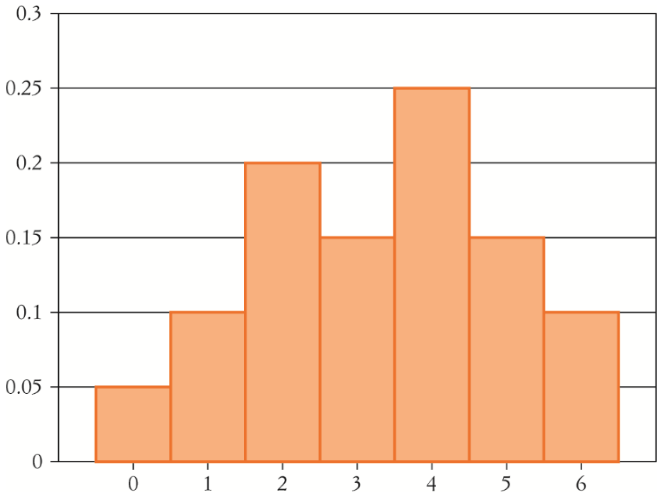
11 Variance 2.92, standard deviation 1.71

12 **a**

x	37	38	39	40	41	42	43
$P(X = x)$	0.02	0.18	0.2	0.4	0.15	0.04	0.01

b **i** $P(x \geq 40) = 0.6$ **ii** $P(x < 39) = 0.2$

13 **a** 0.5 **b** $\frac{7}{20}$ **c**



14 **a** –$1 **b** –$2.50

15 **a**

x	0	1	2
$P(x)$	0.51	0.42	0.07

b Mean 0.56, variance 0.62, standard deviation 0.39
c

y	0	1	2
$P(y)$	7/20	0.5	3/20

Sequences and Series Answers:

1 $T_n = 4n + 3$ **2** C **3** $\frac{2}{3}$ **4** 162, −54, 18, −6

5 8 **6** $\frac{625}{6}$ **7** $400\sqrt{3}$ **8** C

9 a Show
b $S_{50} = 2500x + 3625$

10 T_{40}

11 $400\pi + 16$

12 a Show
b 2
c Show

13 83

14 a $t_1 = \log_2 x$
$t_2 = \log_2 x^2 = 2\log_2 x$
$t_3 = \log_2 x^3 = 3\log_2 x$
Common difference $d = \log_2 x$
b $\log_2 x^{25} = 25\log_2 x$ **c** 50 **d** 650

15 8
16 -460
17 $k = 20$
18 Proof