Name: ____________________

Year 12 2024 Extension 2 Mathematics Assessment Task 3

Investigative Assignment with Validation Task		
Task number: 3	**Weighting:** 25%	**Due Date:** Monday 24/6/24 (8:00am to 9:00am in Rm 106)

Outcomes assessed:

MEX12-1	understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts
MEX12-2	chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings
MEX12-3	uses vectors to model and solve problems in two and three dimensions
MEX 12-5	applies techniques of integration to structured and unstructured problems
MEX12-7	applies various mathematical techniques and concepts to model and solve structured, unstructured and multi-step problems
MEX12-8	communicates and justifies abstract ideas and relationships using appropriate language, notation and logical argument

Nature and description of the task:

As a result of completing this Assignment, students should be familiar with all content related to the following topics:

- 3D Vectors i.e. Chapter 3 of the Extension 2 textbook.
- Further Integration ie. Chapter 6 of the Extension 2 textbook.

On the 25th June, 2024 you will receive a selection of similar questions to the Preparation Activity below to complete in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive in the in-class Validation task. NOTE: You will not have to hand in the answers to the questions in this Preparation Activity AND you will not have access to the Preparation Activity during the Validation Task.

Non-Completion of Task:

If you know you are going to be away on the day of the Validation Task and are unable to hand in / complete both parts of the Assignment on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is submitted/completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Part 1 Preparation Activity – investigate/attempt each of the following questions in preparation for the in-class Validation Task.

3D Vectors:

1 Find the magnitude of $\underset{\sim}{u} = 3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$.

A 2 **B** $\sqrt{6}$

C $\sqrt{14}$ **D** 14

2 Find the unit vector in the opposite direction to $\underset{\sim}{u} = 3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$.

A $\frac{3}{\sqrt{14}}\underset{\sim}{i} + \frac{1}{\sqrt{14}}\underset{\sim}{j} - \frac{2}{\sqrt{14}}\underset{\sim}{k}$

B $\frac{-3}{\sqrt{14}}\underset{\sim}{i} - \frac{1}{\sqrt{14}}\underset{\sim}{j} + \frac{2}{\sqrt{14}}\underset{\sim}{k}$

C $\frac{3}{\sqrt{6}}\underset{\sim}{i} + \frac{1}{\sqrt{6}}\underset{\sim}{j} - \frac{2}{\sqrt{6}}\underset{\sim}{k}$

D $\frac{-3}{\sqrt{6}}\underset{\sim}{i} - \frac{1}{\sqrt{6}}\underset{\sim}{j} + \frac{2}{\sqrt{6}}\underset{\sim}{k}$

3 The position vector $\underset{\sim}{u}$ is represented by $\overline{OP}$ and the position vector $\underset{\sim}{v}$ is represented by $\overline{OQ}$.
Which one of the following expressions represents $\overline{PQ}$?

A $\underset{\sim}{u} - \underset{\sim}{v}$ **B** $-\underset{\sim}{u} - \underset{\sim}{v}$

C $\underset{\sim}{u} + \underset{\sim}{v}$ **D** $\underset{\sim}{v} - \underset{\sim}{u}$

4 Given that $|\underset{\sim}{u}| = 2$ and $|\underset{\sim}{v}| = 3$ and the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is 45°, what is the scalar product $\underset{\sim}{u} \cdot \underset{\sim}{v}$?

A 4.2 **B** 4.5

C 5.1 **D** 6.2

5 What is the scalar product of $\underset{\sim}{u} = 3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} - 2\underset{\sim}{j} + 6\underset{\sim}{k}$?

A –5 **B** –1

C 7 **D** 11

6 Which one of these is the equation of the line joining (2, 1, −1) and (0, 3, 1)?

A $\underset{\sim}{r}=\left(2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)+\lambda\left(3\underset{\sim}{j}+\underset{\sim}{k}\right)$

B $\underset{\sim}{r}=\left(3\underset{\sim}{j}+\underset{\sim}{k}\right)+\lambda\left(2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)$

C $\underset{\sim}{r}=\left(2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)+\lambda\left(-2\underset{\sim}{i}+2\underset{\sim}{j}+2\underset{\sim}{k}\right)$

D $\underset{\sim}{r}=\left(-2\underset{\sim}{i}+2\underset{\sim}{j}+2\underset{\sim}{k}\right)+\lambda\left(-2\underset{\sim}{i}+2\underset{\sim}{j}+2\underset{\sim}{k}\right)$

7 A line has equation $\dfrac{x-1}{3}=\dfrac{y-2}{-2}=\dfrac{z+6}{1}$.

Which of the following is a vector parallel to the line?

A $3\underset{\sim}{i}-2\underset{\sim}{j}+\underset{\sim}{k}$

B $\underset{\sim}{i}-2\underset{\sim}{j}+6\underset{\sim}{k}$

C $\dfrac{1}{3}\underset{\sim}{i}+\underset{\sim}{j}+6\underset{\sim}{k}$

D $-2\underset{\sim}{i}+4\underset{\sim}{j}+5\underset{\sim}{k}$

8 Given the parametric equations of a line, $x=3-5t, y=2+t, z=3-2t$, which of the following is the Cartesian equation of the same line?

A $\dfrac{x+3}{-5}=\dfrac{y+2}{1}=\dfrac{z+3}{-2}$

B $\dfrac{x+5}{3}=\dfrac{y-1}{2}=\dfrac{z+2}{3}$

C $\dfrac{x}{-15}=\dfrac{y}{2}=\dfrac{z}{-6}$

D $\dfrac{x-3}{-5}=\dfrac{y-2}{1}=\dfrac{z-3}{-2}$

9 Find the equation of the line through (2, 1, 2) parallel to the line with equation.
$\dfrac{x-2}{3}=\dfrac{y-1}{1}=\dfrac{z-5}{4}$.

A $\underset{\sim}{r}=\left(2\underset{\sim}{i}+\underset{\sim}{j}+2\underset{\sim}{k}\right)+\lambda\left(2\underset{\sim}{i}+\underset{\sim}{j}+5\underset{\sim}{k}\right)$

B $\underset{\sim}{r}=\left(2\underset{\sim}{i}+\underset{\sim}{j}+2\underset{\sim}{k}\right)+\lambda\left(3\underset{\sim}{i}+\underset{\sim}{j}+4\underset{\sim}{k}\right)$

C $\underset{\sim}{r}=\left(2\underset{\sim}{i}+\underset{\sim}{j}+\underset{\sim}{k}\right)+\lambda\left(3\underset{\sim}{i}+\underset{\sim}{j}+4\underset{\sim}{k}\right)$

D $\underset{\sim}{r}=\left(2\underset{\sim}{i}+\underset{\sim}{j}+\underset{\sim}{k}\right)+\lambda\left(3\underset{\sim}{i}-\underset{\sim}{j}+4\underset{\sim}{k}\right)$

10 Which of the following lines is parallel to $x=3+2t, y=2-t, z=3+4t$?

A $\dfrac{x-2}{3}=\dfrac{y-1}{2}=\dfrac{z-5}{3}$

B $\dfrac{x-2}{1}=\dfrac{y-1}{3}=\dfrac{z-5}{-1}$

C $\dfrac{x-2}{2}=\dfrac{y-1}{-1}=\dfrac{z-5}{4}$

D $\dfrac{x-2}{5}=\dfrac{y-1}{1}=\dfrac{z-5}{7}$

11 How far is the point (3, 4, −5) from the origin?

12 Find, correct to the nearest degree, the angle between the vectors $2\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k}$ and $\underset{\sim}{i}+2\underset{\sim}{j}+2\underset{\sim}{k}$.

13 The vectors $\underset{\sim}{u}=3\underset{\sim}{i}+n\underset{\sim}{j}+\underset{\sim}{k}$ and $\underset{\sim}{v}=\underset{\sim}{i}+(2n-1)\underset{\sim}{j}+(4n-2)\underset{\sim}{k}$ are perpendicular.
Find the values of n.

14 Use vectors to prove that the diagonals of a rhombus are perpendicular.

15 Given 2 non-zero vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ such that $|\underset{\sim}{a}+\underset{\sim}{b}|=|\underset{\sim}{a}-\underset{\sim}{b}|$, find $\underset{\sim}{a}\cdot\underset{\sim}{b}$.

16 Draw a graph represented by the parametric equations $x = 3\sin t$, $y = 3\cos t$, $z = t$, for $0 \le t \le 2\pi$.

17 Consider the line M with vector equation $\underset{\sim}{m}=\left(3\underset{\sim}{i}+\underset{\sim}{j}+4\underset{\sim}{k}\right)+\lambda\left(2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)$.

Find the coordinates of the point P on the line $\underset{\sim}{m}$ that is nearest to the origin and calculate the distance of the line from the origin.

18 Find the value of n for which the vectors $\underset{\sim}{a}=-8\underset{\sim}{i}+17\underset{\sim}{j}+12\underset{\sim}{k}$ and $\underset{\sim}{b}=n\underset{\sim}{i}+20\underset{\sim}{j}-13\underset{\sim}{k}$ are perpendicular.

19 Find the value of n for which the vectors $\underset{\sim}{u}=-24\underset{\sim}{i}-20\underset{\sim}{j}-40\underset{\sim}{k}$ and $\underset{\sim}{v}=6\underset{\sim}{i}+5\underset{\sim}{j}+n\underset{\sim}{k}$ are collinear.

20 Find the distance of the point (0, 2, 3) from the line $\dfrac{x-3}{2}=\dfrac{y-1}{1}=\dfrac{z+1}{2}$.

21 The line L goes through $A(4, 3, -2)$ and is parallel to the line $\dfrac{x-1}{3}=\dfrac{y-2}{-4}=\dfrac{z-3}{-1}$.

$P(m, n, -5)$ lies on the line L. Determine the values for m and n.

22 Find the perpendicular distance between the point (6, 7, 10) and the line that is parallel to the vector $2\underset{\sim}{i}+\underset{\sim}{j}+\underset{\sim}{k}$ and passes through (5, 9, 4).

23 Find the perpendicular distance between the lines $\underset{\sim}{p}=\left(\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k}\right)+\lambda_1\left(2\underset{\sim}{i}+\underset{\sim}{j}+3\underset{\sim}{k}\right)$ and $\underset{\sim}{q}=\left(2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)+\lambda_2\left(3\underset{\sim}{i}-\underset{\sim}{j}-\underset{\sim}{k}\right)$.

24 Consider the line R with vector equation $\underset{\sim}{r}=\left(-2\underset{\sim}{i}+\underset{\sim}{j}+3\underset{\sim}{k}\right)+\lambda\left(-\underset{\sim}{i}-2\underset{\sim}{j}+\underset{\sim}{k}\right)$.

a Find the coordinates of the point P on R that is nearest to the point (−2, 5, 8).

b What is the distance of the line to the point (−2, 5, 8)?

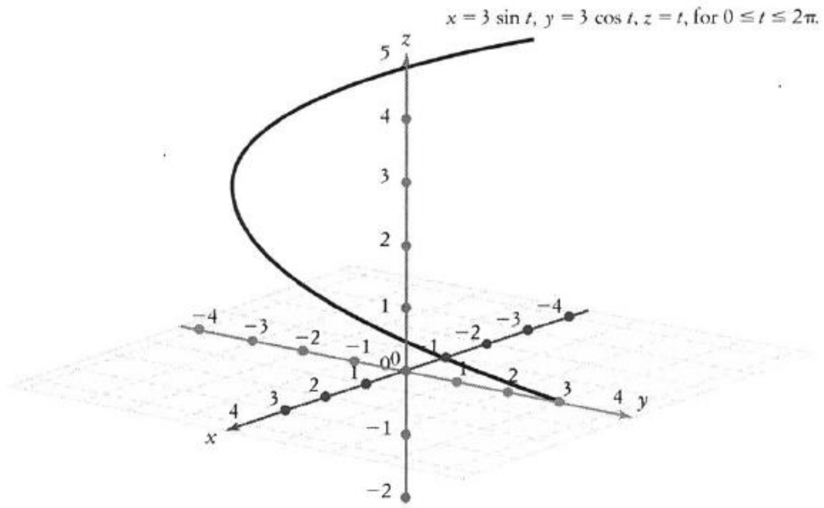
3D Vectors Answers:

1 C **2** B **3** D **4** A **5** A

6 C **7** A **8** D **9** B **10** C

11 $5\sqrt{2}$ **12** 27° **13** $n = -\frac{1}{2}, -1$ **15** 0

16 clockwise spiral, starting at (0, 3, 0) and ending at (0, 3, 2π)



17 $P\left(2, \frac{1}{2}, 4\frac{1}{2}\right), d = \frac{7\sqrt{2}}{2}$ **18** $n = 23$ **19** $n = 10$ **20** 5 **21** $m = 13, n = -9$

22 $\sqrt{35}$ **23** $\frac{\sqrt{6}}{3}$ **24 a** $P\left(-1\frac{1}{2}, 2, 2\frac{1}{2}\right)$ **b** $d = \sqrt{39\frac{1}{2}} \approx 6.28$

Further Integration:

1 Find $\int \frac{1}{9+x^2}dx$.

A $\frac{1}{3}\sin^{-1}\left(\frac{x}{3}\right)+C$ **B** $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

C $3\sin^{-1}(3x)+C$ **D** $3\tan^{-1}(3x)+C$

2 Find real numbers, A and B such that
$\frac{4}{x^2-4}=\frac{A}{x-2}+\frac{B}{x+2}$.

A $A=1, B=1$ **B** $A=1, B=-1$

C $A=2, B=2$ **D** $A=2, B=1$

3 If $f'(x)=\frac{x+1}{(x^2+2x+3)^3}$, what is $f(x)$?

A $\frac{1}{2(x^2+2x+3)^2}+C$ **B** $\frac{1}{4(x^2+2x+3)^2}+C$

C $\frac{-1}{2(x^2+2x+3)^2}+C$ **D** $\frac{-1}{4(x^2+2x+3)^2}+C$

4 If $\int_3^4 \frac{1}{(x-1)(x-2)}dx = 2\ln M$, what is M?

A $\frac{\sqrt{3}}{2}$ **B** $\frac{2}{\sqrt{3}}$ **C** 1 **D** $2\sqrt{3}$

5 Which integral, with an appropriate substitution, is used to find
$\int_1^2 x(2-x)(x^3-3x^2+4)dx$?

A $3\int_1^2 u\,du$ **B** $3\int_2^1 u\,du$

C $\int_2^0 3u\,du$ **D** $\frac{1}{3}\int_0^2 u\,du$

6 Using a suitable substitution, which integral can be used to evaluate $\int_1^{e^2} \frac{(\ln x)^2}{x}dx$?

A $\int_0^2 \frac{u^2}{e^u}du$ **B** $\int_0^{\ln 2} u^2\,du$

C $\int_0^{\ln 2} \frac{u^2}{e^u}du$ **D** $\int_0^2 u^2\,du$

7 Find $\int e^{\sin x}\cos x\, dx$.

A $e^{\sin x} + C$ **B** $e^{\cos x} + C$

C $-e^{\sin x} + C$ **D** $-e^{\cos x} + C$

8 Find $\int \frac{1}{\sqrt{3+2x-x^2}}\, dx$.

A $\frac{1}{2}\sin^{-1}(x-1) + C$ **B** $\frac{1}{2}\sin^{-1}\left(\frac{x-1}{2}\right) + C$

C $\sin^{-1}\left(\frac{x-1}{2}\right) + C$ **D** $\sin^{-1}(x-1) + C$

9 Find $\int x \sin 2x\, dx$.

A $-\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x + C$

B $-\frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x + C$

C $2x \cos 2x + 4 \sin 2x + C$

D $2x \cos 2x - 4 \sin 2x + C$

10 Find $\int \frac{1}{x^3+x}\, dx$.

A $\ln\left|x\sqrt{x^2+1}\right| + C$ **B** $\ln|x(x^2+1)| + C$

C $\ln\left|\frac{x}{\sqrt{x^2+1}}\right| + C$ **D** $\ln\left|\frac{x}{x^2+1}\right| + C$

11 **a** Show that $2x^2 + 12x + 18$ can be written as $2(x+3)^2$.

b Hence, find $\int \frac{5}{2x^2+12x+18}\, dx$.

12 Using $\frac{2x+4}{x^2-4} = \frac{2x}{x^2-4} + \frac{4}{x^2-4}$, show that $\int_3^4 \frac{2x+4}{x^2-4}\, dx = 2\ln 2$.

13 **a** Find real numbers A and B such that $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$.

b Hence, find $\int \frac{1}{x(x+1)}\, dx$.

14 Evaluate $\int_2^4 \frac{x^2-2}{x^2+2}\, dx$.

15 Use integration by parts to evaluate $\int_1^{2e} \ln x$.

16 Use integration by parts to find $\int x \sin x\, dx$.

17 **a** Show that, for the domain $\left(0, \frac{1}{2}\right)$, $\frac{d}{dx}\left(\sin^{-1}\left(\sqrt{2x}\right)\right) = \frac{1}{\sqrt{2x(1-2x)}}$.

b Hence find the exact value of $\int_{\frac{1}{8}}^{\frac{1}{4}} \frac{dx}{\sqrt{2x(1-2x)}}$.

18 Use partial fractions to find $\int \frac{dx}{\left(x^2+3\right)\left(x^2+4\right)}$.

19 Let $I_n = \int x^n \ln x \, dx$.

a Show that $I_n = \frac{x^{n+1}}{(n+1)^2}\left[(n+1)\ln x - 1\right] + C$.

b Hence find I_3.

20 Let $I_n = \int_0^1 \frac{x^n}{1+x^2} dx$ for $n \geq 0$.

a Show that $I_n = \frac{1}{n-1} - I_{n-2}$, for $n \geq 2$.

b Hence find I_4.

21 **a** Evaluate $\int_0^{\frac{\pi}{4}} \tan^4 x \sec^2 x \, dx$.

b Hence, show that $\int_0^{\frac{\pi}{4}} \tan^6 x \, dx = \frac{1}{5} - \int_0^{\frac{\pi}{4}} \tan^4 x \, dx$ and deduce that $\int_0^{\frac{\pi}{4}} \tan^6 x dx = \frac{13}{15} - \frac{\pi}{4}$.

Further Integration Answers:

1 B	**2** B	**3** D	**4** B
5 D	**6** D	**7** A	**8** C
9 A	**10** C		

11 b $\frac{-5}{2(x+3)}+C$

13 a $A=1, B=-1$ **b** $\ln\left|\frac{x}{x+1}\right|+C$

14 $2-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{5}\right)\approx 1.22$

15 $2e\ln 2+1\approx 4.77$

16 $-x\cos x+\sin x+C$

17 b $\frac{\pi}{12}$

18 $-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+C$

19 b $I_3=\frac{x^4}{16}[4\ln x-1]+C$

20 $I_4=\frac{\pi}{4}-\frac{2}{3}$

21 $\frac{1}{5}$

End of Part 1 Preparation Activity

Mathematics Advanced

Mathematics Extension 1

Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \; r \neq 1$$

$$S = \frac{a}{1-r}, \; |r| < 1$$

Logarithmic and Exponential Functions

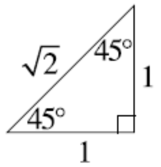
$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

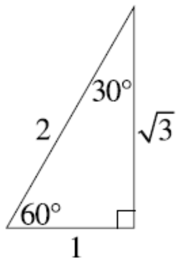
$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$



$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

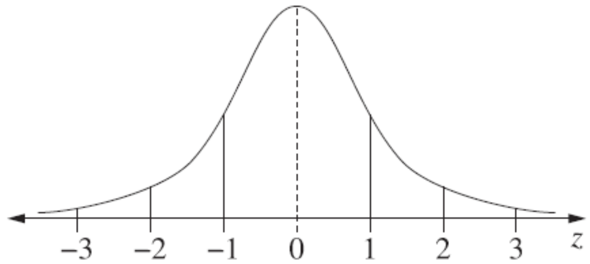
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$