Name: ____________________

Year 12 2024 Mathematics Advanced
Assessment Task 3

Assignment with Validation Task		
Task number: 3	**Weighting:** 25%	**Due Date:** Tuesday 28/5/24

Outcomes assessed:

MA 12-1	uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts
MA 12-3	applies calculus techniques to model and solve problems
MA 12-6	applies appropriate differentiation methods to solve problems
MA 12-7	applies the concepts and techniques of indefinite and definite integrals in the solution of problems
MA 12-8	solves problems using appropriate statistical processes
MA12-10	constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

Nature and description of the task:

As a result of completing this Assignment, students should be familiar with all content related to the following topics:

- Further Differentiation (Chapter 4 of the Advanced Grove Book and Chapter 5 of the Extension 1 Grove Book).
- Geometrical Applications of Differentiation (Chapter 5 of the Advanced Grove Book and Chapter 6 of the Extension 1 Grove Book).
- Integration (Chapter 6 of the Advanced Grove Book and Chapter 7 of the Extension 1 Grove Book).
- Statistics (Chapter 7 of the Advanced Grove Book and Chapter 9 of the Extension 1 Grove Book).

On the 28th May, 2024 you will receive a selection of similar questions to the Preparation Activity below to complete in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive in the in-class Validation task. NOTE: You will not have to hand in the answers to the questions in this Preparation Activity AND you will not have access to the Preparation Activity during the Validation Task.

Non-Completion of Task:

If you know you are going to be away on the day of the Validation Task and are unable to complete it on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Part 1 Preparation Activity – investigate/attempt each of the following questions in preparation for the in-class Validation Task.

Further Differentiation:

1 Find the derivative of each function.

a $f(x) = x^2 - 2x$

b $y = \dfrac{1}{x}$

c $y = \dfrac{x}{x^2 - 1}$

d $g(x) = (x^2 - x)^2$

e $h(x) = 4^x$

f $f(x) = 2xe^{x^2}$

g $f(x) = \log_e x^3$

2 Find the anti-derivative of each function.

a $f(x) = 4x^3 - 3x^2 + x$

b $\dfrac{dy}{dx} = (5x - 7)^3$

c $g(x) = \tan^2 x \sec^2 x$ *<< Extension 1 only*

d $h(x) = \dfrac{4x - 1}{2x^2 - x}$

e $\dfrac{du}{dx} = xe^{x^2}$

f $y' = 2x^3 - 2x \sin x^2 \cos x^2$ *<< Extension 1 only*

3 Find the equation of the normal to the graph for the function $y = x^3 + 2 \ln x^2$ when $x = -1$.

4 A hyperbola has equation $y = \frac{2}{x}$. Find the equation of the tangent to the hyperbola at $x = 2$, in general form.

5 Find the exact gradient of the tangent to the function $f(x) = e^{\ln x^2}$ at $x = \frac{1}{2}$.

6 Find the equation of the function $f(x)$ if $f'(x) = xe^{x^2}$ and the point $P\left(0, \frac{5}{2}\right)$ lies on the graph of the function.

7 Find $y = f(x)$ if $f''(x) = 3x^2 - 6x + 2$, $f'(2) = -3$ and $f(-1) = 2$.

8 Find the derivative of $y = \sin 2x°$.

9 The rate of change of volume V with respect to time (t) is $\frac{dV}{dt} = (2t-1)^3$.

If $V = 3$ when $t = 0$, find the volume when $t = 3$.

10 Find the second derivative of $v = \frac{x}{x^2 - 5}$.

Further Differentiation Answers:

1 **a** $f'(x) = 2x - 2$ **b** $y' = -\frac{1}{x^2}$ **c** $y' = -\frac{x^2+1}{(x^2-1)^2}$

d $g'(x) = 2(x^2 - x)(2x - 1)$ **e** $h'(x) = 4^x \ln 4$ **f** $f'(x) = 2e^{x^2}(2x^2+1)$

g $f'(x) = \frac{3}{x}$

2 **a** $F(x) = x^4 - x^3 + \frac{x^2}{2} + c$ **b** $y = \frac{(5x-7)^4}{20} + c$ **c** $G(x) = \frac{\tan^3 x}{3} + c$

d $y = \ln|2x^2 - x| + c$ **e** $u = \frac{e^{x^2}}{2} + c$ **f** $y = \frac{x^4}{2} + \frac{\cos^2(x^2)}{2}$

3 $y = x$

4 $x + 2y - 4 = 0$

5 $m = 1$

6 $\frac{e^{x^2}}{2} + 2$

7 $f(x) = \frac{x^4}{4} - x^3 + x^2 - 3x - \frac{13}{4}$

8 $y' = \frac{\pi}{90}\cos\left(\frac{\pi x°}{90}\right)$

9 81

10 $\frac{2x^3 + 30x}{(x^2-5)^3}$

Geometrical Applications of Differentiation:

1 For what values of x is the graph of $y = 3x^2 - 4x + 1$ increasing?

2 The curve $f(x) = x^3 + ax^2 + bx - 4$ has stationary points at $x = 3$ and $x = 5$. Find the values of a and b.

3 Draw a sketch of a function $y = f(x)$ where $\frac{dy}{dx} > 0$ for $x < 5$, $\frac{dy}{dx} = 0$ when $x = 5$ and $\frac{dy}{dx} < 0$ for $x > 5$.

4 Consider the function $f(x) = 2x^3 + 6x^2 - 1$ in the domain $[-3, 1]$.

a State the y-intercept.

b Locate any stationary points and determine their nature.

c Determine the point of inflection.

d Hence sketch the graph of $f(x) = 2x^3 + 6x^2 - 1$ in the domain $[-3, 1]$.

e What is the global maximum value of the function in this domain?

5 Consider the graph of the function $y = 2x^3 - 6x^2 + 6x + 1$.

a Show that the graph has only one stationary point, find its coordinates and determine its nature.

b State the values of x for which the curve is concave up.

c State the values of x for which the curve is increasing.

6 Consider the function $y = x\sqrt{4 - x^2}$.

a What is the domain of this function?

b Find the stationary points for this function and determine their nature.

c Sketch the graph of the function.

7 The 3 dimensions (length, breadth and height) of a box with a square base of side x all add up to 40 cm.
Show that the volume of this box is given by $V = 40x^2 - 2x^3$ and hence find the maximum volume of this box.

8 A scenic helicopter flight can take x passengers per week for a charge of $(200 - 0.3x)$ dollars per person. The number of flights is not relevant. The cost of operation each week is $\$(3000 + 50x)$ dollars when x passengers are taken.

a Show that the profit, in dollars, per week is given by $P = 150x - 0.3x^2 - 3000$.

b How many passengers should be taken, per week, to maximise the profit?

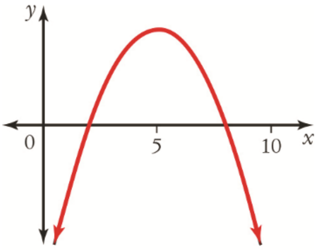
c Hence, given this number of passengers, what is the charge, per passenger, when this maximum profit occurs?

Geometrical Applications of Differentiation Answers:

1 $x > \frac{2}{3}$

2 $a = -12, b = 45$

3

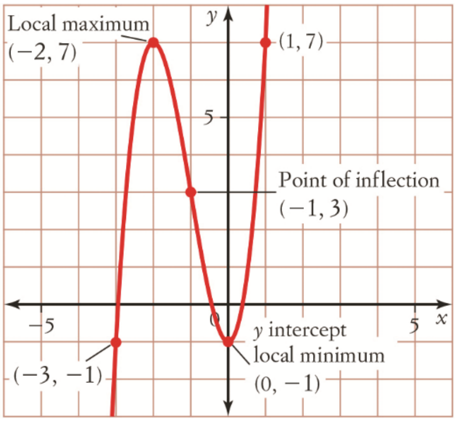


4 **a** -1

b $(-2, 7)$ maximum, $(0, -1)$ minimum

c $(-1, 3)$

d



e $y = 7$

5 **a**

$$\frac{dy}{dx} = 6x^2 - 12x + 6$$
$$= 6(x^2 - 2x + 1)$$
$$= 6(x-1)^2$$

Stationary point occurs when $\frac{dy}{dx} = 0$

$6(x-1)^2 = 0$

$\therefore x = 1$

$$\frac{d^2y}{dx^2} = 12x - 12$$
$$= 12(x-1)$$

When $x = 1$, $\frac{d^2y}{dx^2} = 0$

x	0	1	2
y''	−12	0	12

Hence one stationary point is a horizontal point of inflection (1, 7).

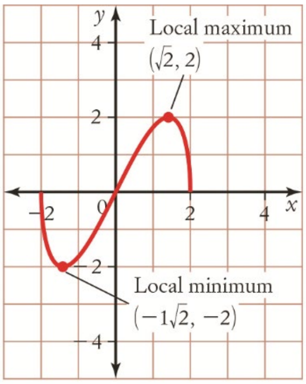
b $x > 1$

c all x except $x = 1$

6 **a** $[-2, 2]$

b $\left(-\sqrt{2}, -2\right)$ mimimum, $\left(\sqrt{2}, 2\right)$ maximum

c



7 $2x + h = 40$

$h = 40 - 2x$

$V = x^2 h$

$= x^2(40 - 2x)$

$= 40x^2 - 2x^3$

$\frac{dV}{dx} = 80x - 8x^2$

$= 8x\,(10 - x)$

$= 0$ when $x = 0, 10$

However 0 is not a solution

$\frac{d^2V}{dx^2} = 80 - 16x$

$f''(10) = 80 - 160$

$= -80\ (< 0)$

$\therefore$ Max volume occurs when $x = 10$

Max volume = $10^3 = 1000\ \text{cm}^3$

8 **a** Weekly Profit = Income − Expenditure

= charge × no. of passengers

− costs of operation

$P = (200 - 0.3x)x - (3000 + 50x)$

$= 150x - 0.3x^2 - 3000$

b 250 **c** \$125

Integration:

1 Use the trapezoidal rule to approximate $\int_2^5 \frac{1}{x-1}\,dx$ using 4 subintervals.

2 Evaluate:

a $\int_1^2 \frac{3x^3 - x^2 + 4x}{x}\,dx$

b $\int_2^3 (4x+2)\sqrt{x^2+x-4}\,dx$

3 Find the change in displacement between 1 and 3 seconds if the velocity function is $v = 6t^2 - 8t + 1$ cm s^{-1}.

4 Find each integral.

a $\int \sqrt{x} + \frac{2}{x^2} + e^{3-x}\,dx$

b $\int 3x\sec^2(3x^2)\,dx$

c $\int -\sin x\,dx$

5 **a** Show that $\frac{3x-4}{x^2-16} = \frac{1}{x-4} + \frac{2}{x+4}$.

b Hence find $\int \frac{3x-4}{x^2-16}\,dx$.

6 A particle has velocity $v = \frac{t}{t^2+1}$ m s^{-1}. Find its displacement after 4 s if its initial displacement is 3 m.

7 A curve has its rate of change given by $\frac{dy}{dx} = \sin(3x)$ and passes through the point $\left(\frac{\pi}{2}, \pi\right)$.

Find the equation of the curve.

8 Find the area enclosed by the curve $y = e^{3^x} - 1$ and the lines $y = 0$ and $x = 1$.

9 Find the area enclosed between the curves $y = x^2$, $y = (x+2)^2$ and the x-axis.

10 **a** Sketch and shade the area bounded by the curve $y=\sqrt{x+4}$, the y-axis and the lines $y = 0$ and $y = 3$.

b Calculate the total shaded area.

Integration Answers:

1 1.42809 units^2

2 **a** 9.5 **b** $\frac{56\sqrt{2}}{3}$

3 **a** 23 cm

4 **a** $\frac{2\sqrt{x^3}}{3}-\frac{2}{x}-e^{3-x}+C$ **b** $\frac{1}{2}\tan\left(3x^2\right)+C$ **c** $\cos x$ ⍰ C

5 **a**

$$\begin{aligned}\text{RHS} &= \frac{1}{x-4}+\frac{2}{x+4}\\ &= \frac{1(x+4)+2(x-4)}{(x-4)(x+4)}\\ &= \frac{x+4+2x-8}{x^2-16}\\ &= \frac{3x-4}{x^2-16}\\ &= \text{LHS}\end{aligned}$$

b $\ln|x-4| + 2\ln|x+4| + C$

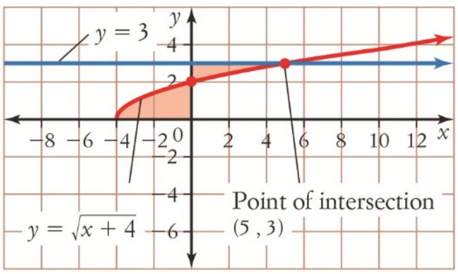
6 7.4166 m

7 $y=-\frac{1}{3}\cos(3x)+\pi$

8 $\frac{1}{3}e^3-\frac{4}{3}$ units^2

9 $\frac{2}{3}$ units^2

10 **a**



b Area = $7\frac{2}{3}$ units^2

Statistics:

1 The stem-and-leaf plot displays the ages of people attending a gym session one morning.

Stem	Leaf
1	6 8
2	0 1 2 4 5
3	2 5 5 5
4	1 2 6
5	5 7 9

Find each statistic.

a Range

b Mode

c Median

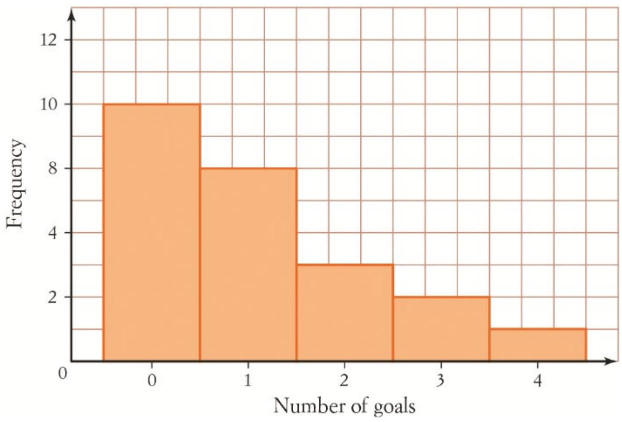
d Interquartile range

e Mean, correct to 2 decimal places

f Population standard deviation, correct to 2 decimal places

2 A group of 8 students completed a test. The mean for the group on this test was 65. However, one of the scores was incorrectly recorded as a 50 instead of 60. What is the correct mean?

3 The number of goals scored by a hockey team throughout each match in the season is displayed in the frequency histogram below.



a How many matches did this hockey team play in the season?

b What is the mode for the number of goals scored per match?

c Calculate the mean number of goals scored per match.

d Describe the skewness of this data.

4 A group of 180 people were surveyed on how far they lived from school.

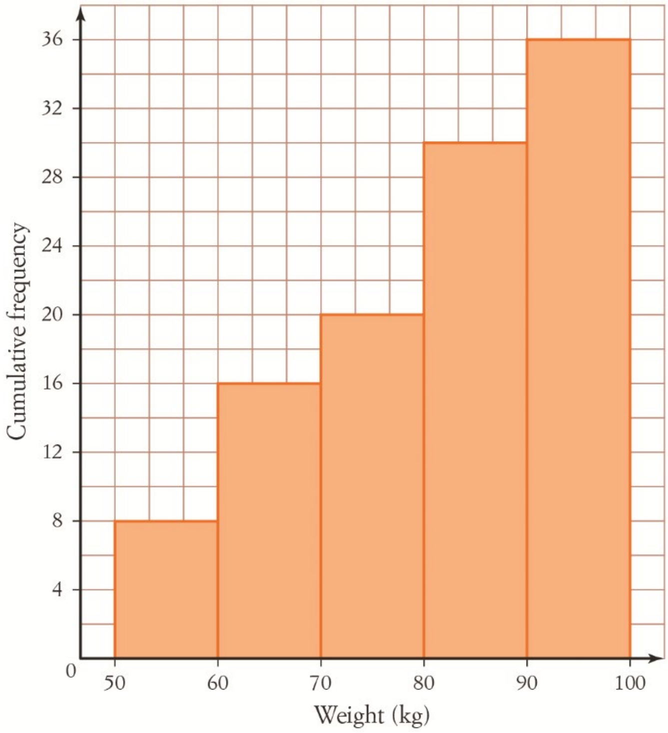
A two-way table is used to record the results.

	Boys	Girls
Under 5 km	65	52
5 km or more	A	29

a What is the value of **A**?

b If a boy is selected at random, what is the probability that he lives less than 5 km from school?

5 The masses of 36 boxers at the local gym were recorded and grouped into class intervals and then displayed in the cumulative frequency graph shown.



a Draw the cumulative frequency polygon (ogive) on the diagram above.

b Use the ogive and estimate the median and the interquartile range.

c Given that the lightest weight of a boxer was 52 kg and the heaviest boxer was 99 kg, draw a box plot for this data.

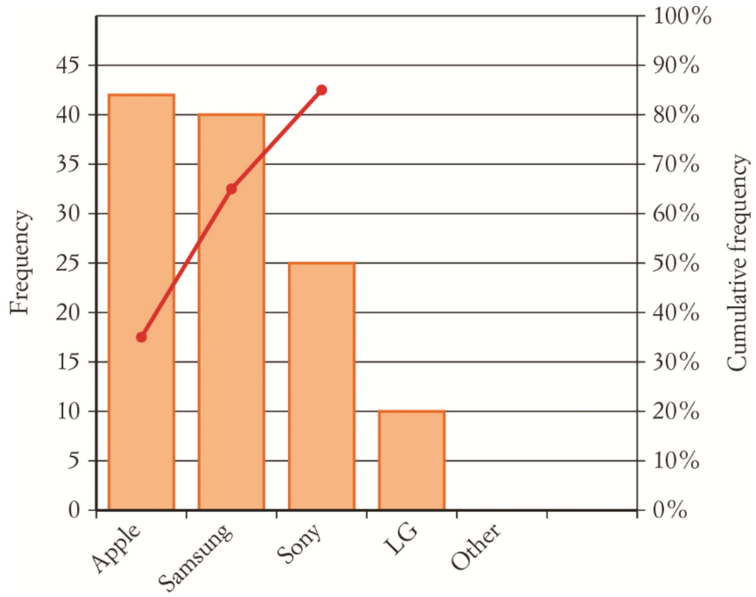
d Estimate the 9th decile.

6 A group of students were asked the brand of mobile phone they owned.

a Complete the percentage frequency and cumulative percentage frequency columns.

Brand	Frequency	Percentage frequency	Cumulative percentage frequency
Apple	42	33.6%	33.6%
Samsung	40	32%	65.6%
Sony	25	20%	
LG	10		
Other	8		100%

b Complete the column graph and the cumulative percentage frequency on the Pareto chart below.



7 At a small supermarket, the 10 employees earn the following wages per week:

$400, $450, $460, $495, $520, $545, $545, $545, $610, $990

a Is the wage of $990 an outlier for this set of data? Justify your answer with calculations.

b Each employee receives a $50 increase. What effect will this have on the standard deviation?

8 Lily's scores in her 5 maths quizzes were: 7, 5, 3, 10, 5

a Calculate the mean of Lily's quizzes, $\bar{x}$

b Complete the table to calculate the variance, σ^2.

Score, x	$x - \bar{x}$	$(x - \bar{x})^2$
3		
5		
5		
7		
10		

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{n}$$

$=$

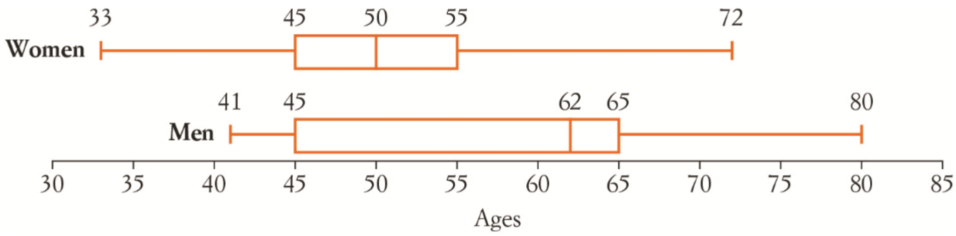
9 The points scored by 2 basketball players over their last 6 games were recorded.

a Complete the table.

Player	Points scored	Mean, $\bar{x}$	Standard deviation, σ
Ajay	10, 11, 12, 12, 18, 18	13.5	
Benny	2, 8, 11, 13, 18, 29		

b Explain who is the more consistent point-scorer per game.

10 The five-number summaries of the ages of the men and women who attend a dance class is displayed in the box plots below.



Compare and contrast the 2 data sets by referring to the summary statistics, spread and the shape.

Statistics Answers:

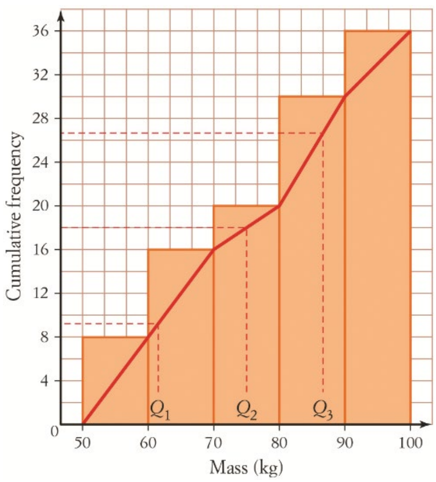
1 a 53 **b** 35 **c** 35 **d** 20 **e** 34.29 **f** 13.56

2 66.25

3 a 24 **b** 0 **c** 1 **d** Positively skewed

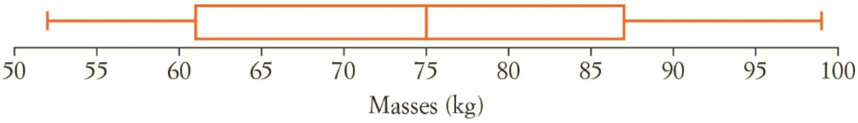
4 a 34 **b** $\frac{65}{99}$

5 a



b Median $Q_2 = 75$, IQR $= Q_3 - Q_1 = 87 - 61 = 26$

c



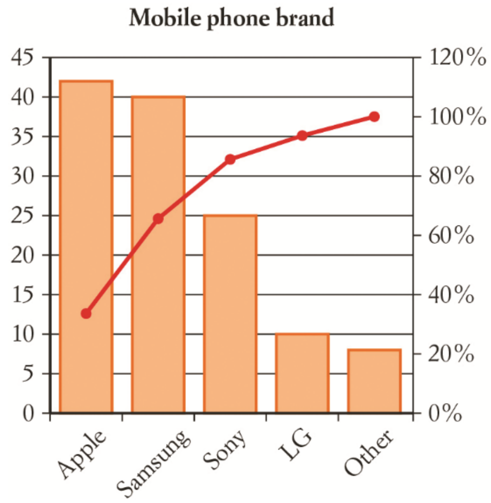
d 93 or 94 kg

6 a

Brand	Frequency	Percentage frequency	Cumulative percentage frequency
Apple	42	33.6%	33.6%
Samsung	40	32%	65.6%
Sony	25	20%	85.6%
LG	10	8%	93.6%
Others n	8	6.4%	100%

Total 125

b



7 a $Q_3 + 1.5 \times \text{IQR} = \$545 + 1.5 \times 85 = \$672.50$

$990 is greater and hence is an outlier wage.

b No change in standard deviation.

8 a $\bar{x} = 6$

b

Score, x	$x - \bar{x}$	$(x - \bar{x})^2$
3	−3	9
5	−1	1
5	−1	1
7	1	1
10	4	16

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{n} = \frac{28}{5} = 5.6$$

9 a

Player	Mean, x	Standard deviation, σ
Ajay	13.5	3.25
Benny	13.5	8.46
Ajay	13.5	3.25
Benny	13.5	8.46

b Ajay is the most consistent as his scores had the lowest spread, with his range and standard deviation being much less than Benny's.

10 In the dance class, the men are generally older.

- Men have the oldest age of 80, while the women have the youngest age of 33.
- The range of 39 years is the same for both the men and women, however the interquartile range of women is lower at 10 years, while the men is 20 years.
- The median age of the men is higher at 62, while the women is 50.
- The women's distribution of ages is symmetrical whereas the men's ages are negatively skewed.

End of Part 1 Preparation Activity

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

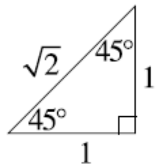
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

$A = \dfrac{1}{2}ab\sin C$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

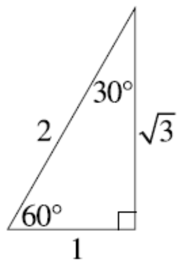


$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$



Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$\cos A = \dfrac{1 - t^2}{1 + t^2}$

$\tan A = \dfrac{2t}{1 - t^2}$

$\cos A\cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

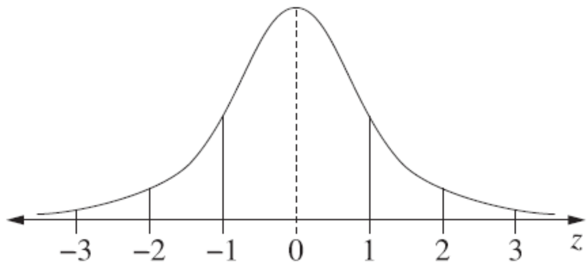
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq x) = \displaystyle\int_a^x f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1 - p)^{n - r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \dbinom{n}{x} p^x (1 - p)^{n - x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$