Name: ____________________

Year 12 2024 Mathematics Extension 1 Assessment Task 3

Assignment with Validation Task

Task number: 3	**Weighting:** 25%	**Due Date:** Wednesday 12/6/24

Outcomes assessed:

ME 12-1 applies techniques involving proof or calculus to model and solve problems
ME 12-2 applies concepts and techniques involving vectors and projectiles to solve problems
ME 12-3 applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations
ME 12-4 uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution
ME 12-7 evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

Nature and description of the task:

As a result of completing this Assignment, students should be familiar with all content related to the following topics:

- Vectors (Chapter 3 of the Extension 1 Grove Book).
- Trig Identities and Further Trig Equations (Ex 4:04 and Ex 4:05 of the Extension 1 Grove Book).
- Derivatives and Integrals of Inverse Functions including Inverse Trig Functions (Ex 5:09, Ex 5:10 and Ex 8:05 of the Extension 1 Grove Book).
- Further Integration (Chapter 8 of the Extension 1 Grove Book).
- Further Vectors (Chapter 10 of the Extension 1 Grove Book).
- Differential Equations (Chapter 13 of the Extension 1 Grove Book).

On the 12th June, 2024 you will receive a selection of similar questions to the Preparation Activity below to complete in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive in the in-class Validation task. NOTE: You will not have to hand in the answers to the questions in this Preparation Activity AND you will not have access to the Preparation Activity during the Validation Task.

Non-Completion of Task:

If you know you are going to be away on the day of the Validation Task and are unable to complete it on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Part 1 Preparation Activity – investigate/attempt each of the following questions in preparation for the in-class Validation Task.

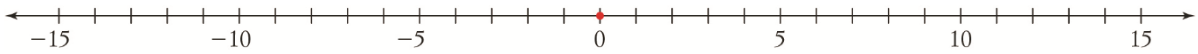
Vectors:

1 State whether each quantity is a scalar or vector.

- **a** temperature
- **b** gravity
- **c** mass
- **d** earthquake intensity
- **e** magnetic force

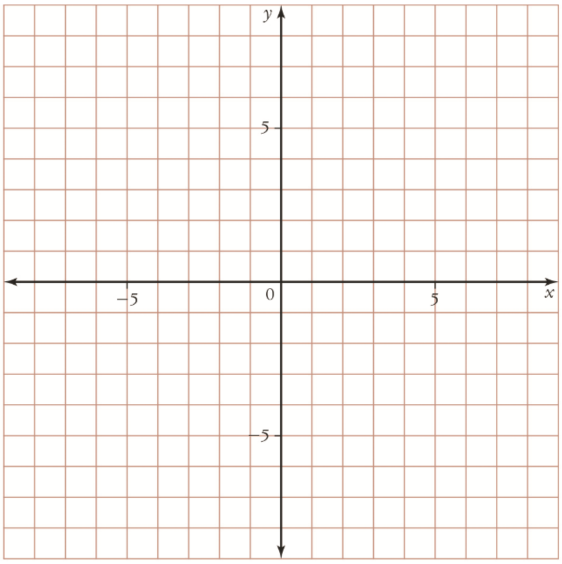
2 An object starts at the origin. It moves for 2 s with a velocity of 5 m s^{-1}, moves for 5 s with velocity –3 m s^{-1} and then moves for 4 s with a velocity of 4 m s^{-1}.

- **a** Complete this diagram, showing the motion of the object.



- **b** What is the final displacement of the object?
- **c** What is the total distance it moves?

3 **a** Draw $\overrightarrow{OA}=\begin{pmatrix}-2\\-3\end{pmatrix}$, $\overrightarrow{OB}=\begin{pmatrix}5\\2\end{pmatrix}$ and $\overrightarrow{AB}$ on the number plane.



b Write $\overrightarrow{AB}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

c What is the position vector equal to $\overrightarrow{AB}$?

d What is the magnitude and direction of $\overrightarrow{AB}$?

4 If an object is subject to forces $9\underset{\sim}{i}$ newtons and $12\underset{\sim}{j}$ newtons, what is the magnitude of the resultant force in newtons?

5 Vector $\underset{\sim}{v}=x\underset{\sim}{i}+y\underset{\sim}{j}$ has magnitude 4 units and direction 240°. Find the exact values of x and y.

6 $\underset{\sim}{a}=-5\underset{\sim}{i}+3\underset{\sim}{j}$ and $\underset{\sim}{b}=2\underset{\sim}{i}-\underset{\sim}{j}$.

a Find $\underset{\sim}{a}+2\underset{\sim}{b}$.

b Rewrite $\underset{\sim}{a}+2\underset{\sim}{b}$ in the form $\begin{pmatrix}x\\y\end{pmatrix}$.

7 A train travels 3 km due north and then 5 km northeast.

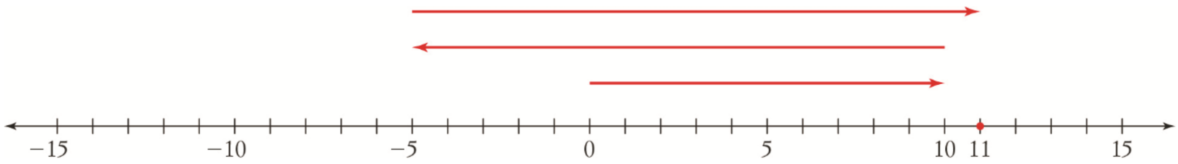
a Represent these displacements graphically and show the resultant displacement.

b Determine the resultant displacement and write it in the form (r cos θ, r sin θ), where r is the magnitude and θ is the direction.

Vectors Answers:

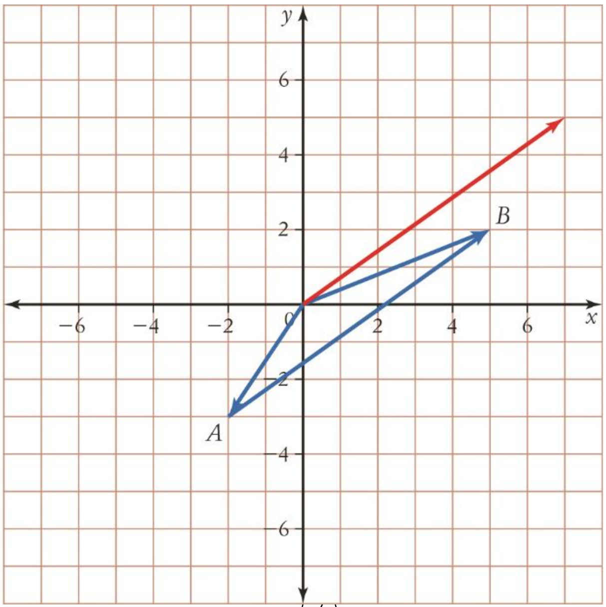
1 **a** scalar **b** vector **c** scalar **d** scalar **e** vector **f** vector

2 **a**



b 11 m **c** 41 m

3 **a**



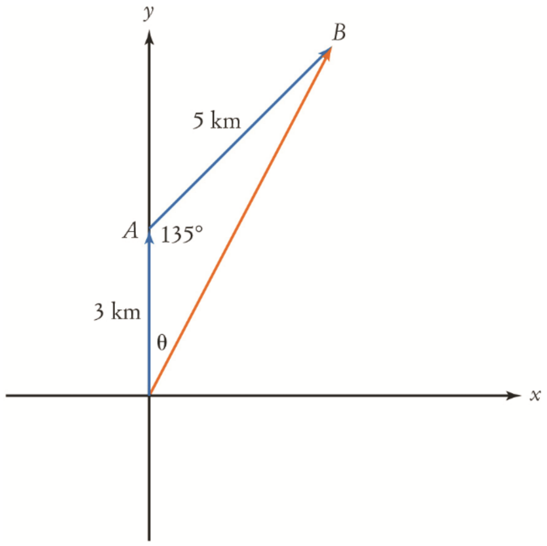
b $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ **c** $\begin{pmatrix} 7 \\ 5 \end{pmatrix}$ **d** $\sqrt{74}$ units, 35°329

4 15 N

5 $x = -2$ $y = -2\sqrt{3}$

6 **a** $-\underset{\sim}{i} + \underset{\sim}{j}$ **b** $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

7 **a**



b (7.4 cos 28°, 7.4 sin 28°)

Trig Identities and Further Trig Equations:

- Refer to Extension 1 Grove Book Practice Set 1 from p 178 Questions 13, 21c and d, 23, 36c, 39, 52, 55

Derivatives and Integrals of Inverse Functions including Inverse Trig Functions:

- Refer to Extension 1 Grove Book Practice Set 2 from p 371 Questions 8, 9, 10, 16, 17, 27, 28, 33, 34, 39, 49 and 53.

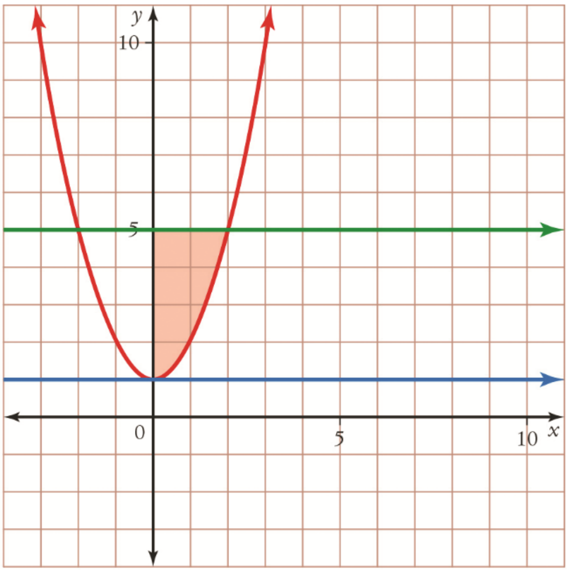
Further Integration:

1 Sketch $f(x) = (x + 2)(x - 1)(x - 3)$ and calculate the physical area cut off by the curve and the x-axis.

2 Find the exact volume of the solid formed when the curve $y = e^{-x}$ is rotated about the x-axis from $x = 0$ to $x = 2$.

3 Use the substitution $u = x - 2$ to evaluate $\int_2^3 x\sqrt{x-2}\,dx$.

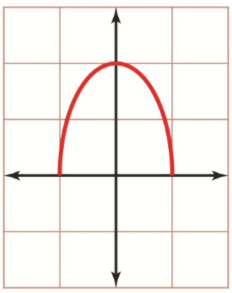
4



a Find the area of the shaded region between the function $y = x^2 + 1$, the y-axis and the lines $y = 1$ and $y = 5$.

b Find the volume of the solid of revolution formed when $y = x^2 + 1$ is rotated about the y-axis from $y = 1$ to $y = 5$.

5 The diagram shows the curve $y = 2\sqrt{4-x^2}$.



a What are the x-intercepts of this curve?

b Calculate the volume of the solid formed when the region enclosed by this curve and the x-axis is rotated around the x-axis.

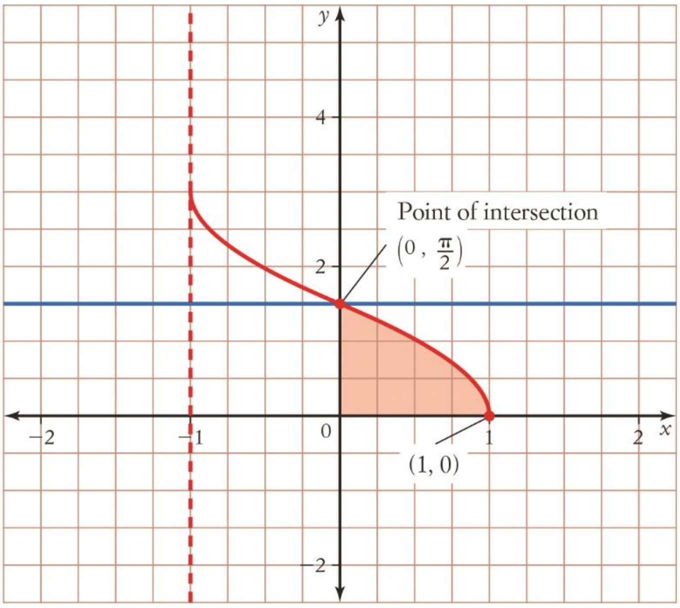
c What is the y-intercept of this curve?

d Calculate the volume of the solid formed when the region enclosed by this curve and the x-axis is rotated around the y-axis.

6 **a** Sketch the curve $y = \ln x$.

b The curve $y = \ln x$ between $x = 1$ and $x = e$ is rotated about the y-axis.
Find the exact value of the volume of the solid formed.

7 A solid is formed by rotating the region bounded by the curve $y = \cos^{-1} x$, the positive x-axis and the line $y = \frac{\pi}{2}$ about the y-axis. Find the volume of this solid.

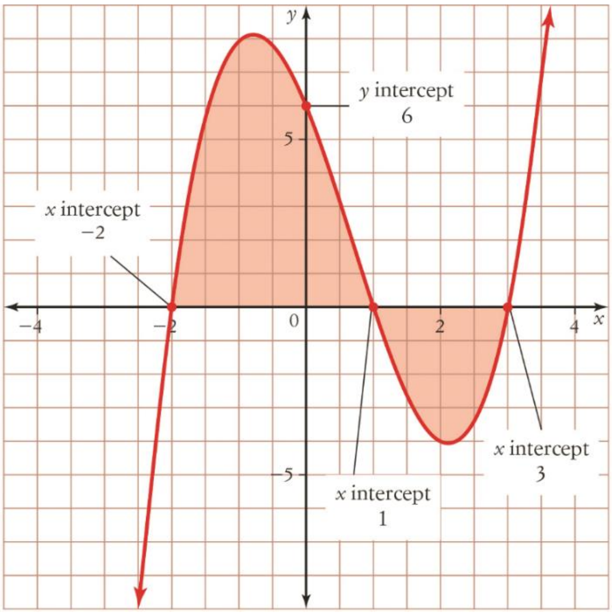


8 **a** Prove that $\cos 3x \ \cos 2x = \frac{1}{2}\left[\cos(3x - 2x) + \cos(3x + 2x)\right]$.

b Hence evaluate $\int_0^{\frac{\pi}{2}} \cos 3x \cos 2x \, dx$.

Further Integration Answers:

1



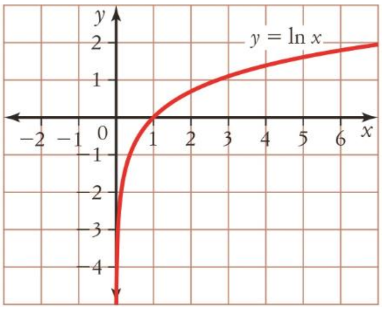
2 $\frac{\pi}{2}(1-e^{-4})$ units^3

3 $\frac{26}{15}=1\frac{11}{15}$

4 **a** $\frac{16}{3}$ units^2 **b** 8π units^3

5 **a** $x = \pm 2$ **b** $\frac{128\pi}{3}$ units^3 **c** 4 **d** $\frac{32\pi}{3}$ units^3

6 **a**



b $\frac{\pi}{2}(e^2-1)$

7 $\frac{\pi^2}{4}$ units^3

8 $\cos 3x \cos 2x = \frac{1}{2}\left[\cos(3x-2x)+\cos(3x+2x)\right]$

a $\cos(3x - 2x) + \cos(3x + 2x) = \cos 3x \cos 2x + \sin 3x \sin 2x + \cos 3x \cos 2x - \sin 3x \sin 2x$

$= 2\cos 3x \cos 2x$

$\therefore \frac{1}{2}[\cos(3x - 2x) + \cos(3x + 2x)] = \cos 3x \cos 2x$

b $\frac{3}{5}$

Further Vectors:

11 a Write the vectors $u = (-3\cos 45°, 2\sin 45°)$ and $v = (2\cos 60°, -3\sin 60°)$ in the form $ai + bj$.

b Hence find $u \cdot v$, correct to 2 decimal places.

c Hence find the angle θ between u and v, correct to the nearest degree.

12 State whether each pair of vectors are perpendicular, parallel or neither. If parallel, state whether the direction is like or unlike.

a

$$\underset{\sim}{u} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}, \underset{\sim}{v} = \begin{pmatrix} -12 \\ 3 \end{pmatrix}$$

b $u = (-3\cos 26°, -3\sin 26°), v = (-5\cos 64°, 5\sin 64°)$

c $u = (-2\cos 30°, 2\sin 30°), v = (7\cos 30°, -7\sin 30°)$

d $u = 2i + 4j, v = 5i + 10j$

13

Find $\text{proj}_{\underset{\sim}{u}}\underset{\sim}{v}$ for each pair of vectors.

a $u = (-2\cos 60°, 2\sin 60°)$, $v = (3\cos 135°, -3\sin 135°)$

b $u = -3i - 2j$, $v = 3i + 4j$

c

$\underset{\sim}{u} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \underset{\sim}{v} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

14

By letting $\underset{\sim}{u} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\underset{\sim}{v} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ and $\underset{\sim}{w} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$, prove that $\underset{\sim}{u} \cdot (\underset{\sim}{v} + \underset{\sim}{w}) = \underset{\sim}{u} \cdot \underset{\sim}{v} + \underset{\sim}{u} \cdot \underset{\sim}{w}$.

15

Use vectors to prove that the quadrilateral *ABCD* is a parallelogram if the diagonals bisect each other.

16 A triangle *ABC* has $CA = a$, $CB = b$ and *M* is the midpoint of *AB*. Illustrate this information with a diagram, then show that:

a $BA = a - b$

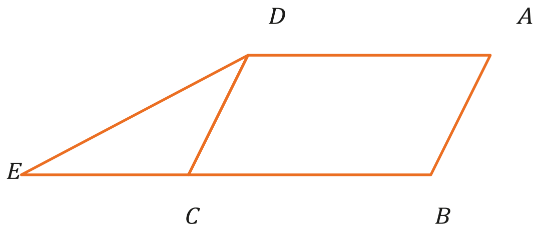
b

$\overrightarrow{MB} = -\frac{1}{2}(a - b)$

c

$\overrightarrow{CM} = \frac{1}{2}(a + b)$

17 $ABCD$ is a parallelogram where $DA = a$, $DB = b$ and $EC : CB = 2 : 3$.



Show that:

a $\overrightarrow{AB} = b - a$

b $\overrightarrow{CD} = a - b$

c $\overrightarrow{BE} = -\frac{5}{3}a$

d $\overrightarrow{EA} = \frac{8}{3}a - b$

18 Johanna is participating in a model boat race. To practise, she went to the river. She places the boat directly opposite her friend Peter who is standing on the other bank waiting for it. The boat's velocity is 83 m/min and the current of the river affecting the boat is 133 m/min west.

a What will be the resultant speed of the boat influenced by the current?

b Will the boat reach Peter? If not, how far away will the boat be from Peter if the river is 150 m wide?

19 James is standing on a hill 20 m above the ground and practising to hit a target with his bow and arrow 25 m away from the base of the hill. The arrow leaves the bow with a speed of 25 m/s at an angle of 35° with the horizontal.

c Using the origin for the base of the hill, derive the vector equation for the arrow's displacement (let $g = 10\text{ m s}^{-2}$).

d Derive the equation of the arrow's flight as a function *y* in terms of *x*.

e Calculate the time for the arrow to reach the ground.

f Will the arrow hit the target? Explain your answer.

Further Vectors Answers:

11 a $u = -\frac{3}{\sqrt{2}}\underset{\sim}{i} + \frac{2}{\sqrt{2}}\underset{\sim}{j}$; $v = \underset{\sim}{i} - \frac{3\sqrt{3}}{2}\underset{\sim}{j}$ **b** −5.80 **c** 145°

12 a perpendicular **b** perpendicular

c parallel unlike direction **d** neither

13 a $\frac{3\sqrt{2}+3\sqrt{6}}{8}\underset{\sim}{i} - \frac{3\sqrt{6}+9\sqrt{2}}{8}\underset{\sim}{j}$ **b** $-\frac{45}{13}\underset{\sim}{i} - \frac{60}{13}\underset{\sim}{j}$ **c** $\begin{pmatrix} \frac{32\sqrt{10}}{10} \\ \frac{96\sqrt{10}}{10} \end{pmatrix}$

14–17 Teacher to check

18 a 157 m/min **b** 240 m west

19 a $\underset{\sim}{s} = 21t\underset{\sim}{i} + \left(-5t^2 + 14t + 20\right)\underset{\sim}{j}$ **b** $y = -\frac{5}{441}x^2 + \frac{2}{3}x + 20$

c $t = 1.04$ s **d** No, distance = 21.9 m

Differential Equations:

1 Determine the solution to $\frac{dy}{dx} = 1 - e^{-x}$ that passes through the point (0, 6).

2 Draw the direction field for the differential equation $\frac{dy}{dx} = xy$ and show the solution that passes through *A*(0, 1).

3 Solve the differential equation $\frac{dy}{dx} = \sin^3 x \cos^3 x$ given that $y(0) = 0$.

4 Solve the differential equation $\frac{dy}{dx} = y^2 + 1$, given that $y(0) = 1$.

5 A glass of water at a temperature of 20°C is placed in a refrigerator. The rate at which the water's temperature decreases is proportional to the difference between its temperature and the 4°C temperature inside the refrigerator. When the glass has been in the refrigerator for 10 minutes, the temperature of the water in the glass is 12°C. What is the temperature of the water in the glass after
20 minutes?

6 The rate of decay of a radioactive substance is proportional to the amount, *M*, of the substance present at any time, given by the differential equation $\frac{dM}{dt} = -kM$, where *k* is a constant.

Initially, $M = 20$ units, and when $t = 20$ hours, $M = 5$ units.

How long does it take for *M* to decay to 2 units?

7 After use of an insecticide, the insect population is given by $N = 200 - \frac{150}{1+e^{-t}}$, where *N* is the number of insects and *t* is the time in hours after the insecticide has been applied.

a How many insects were there initially?

b Write an expression for the rate at which the population of insects is changing.

c Using this model, will the population disappear altogether?

d How long will it take for the population of insects to diminish to 100?

8 The rate at which a virus spreads throughout a population of 1000 animals in a specific area is proportional to the product of the number *N* of infected animals and the number of animals not infected after *t* days. Initially, just 2 animals are infected, and after 10 days, 12 are infected.

a Write a differential equation to show the rate at which the virus is spreading.

b Determine how many animals were NOT infected after 30 days.

9 In Newton's Law of Cooling, the rate at which an object changes temperature is proportional to the difference between its temperature and the temperature of the surrounding medium. A lamb roast is cooked in an oven at 200°C. Then it is placed on a cooling rack in a room where the temperature is 30°C. After 5 minutes, the roast has cooled to 120°C.

a Find the temperature after 15 minutes.

b Determine when the roast reaches 60°C.

10 Suppose that t hours after the start of an epidemic in a small community, the number of residents who have caught the disease is given by the equation,

$$P(t) = \frac{1200}{6 + 294e^{-0.8t}}$$

a How many residents had the disease when the epidemic began?

b Approximately how many residents will contract the disease?

c What is the differential equation for $P(t)$?

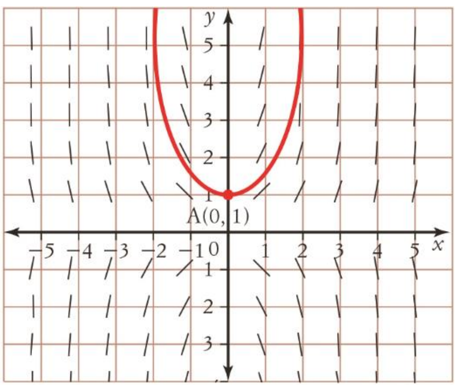
d When was the disease the most contagious?

e How fast was the disease spreading at the peak of the epidemic?

Differential Equations Answers:

1 $y = x + e^{-x} + 5$

2 Solution that passes through $A(0, 1)$ shown in red.



3 $y = \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x$, or $y = -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + \frac{1}{12}$

4 $y = \tan\left(x + \frac{\pi}{4}\right)$

5 $T = 8$°C

6 33.2 hours

7 **a** 125 insects **b** $\frac{dN}{dt} = -\frac{150e^{-t}}{(1+e^{-t})^2}$

c No, as $t \to \infty$, $N \to 50$. **b** 0.69 h or 42 min

8 **a** $\frac{dN}{dt} = kN(1000 - N)$ **b** $1000 - N = 692$ not infected after 30 days

9 **a** $T = 55.2$°C **b** $t = 13.6$ minutes

20 **a** 4

b 200

c $\frac{dP}{dt} = \frac{7840e^{0.8t}}{\left(e^{0.8t} + 49\right)^2}$

d $\frac{d^2P}{dt^2} = 0$, hence $t = 4.86$ hours

e $\frac{dP}{dt}(t = 4.86) = 40$ students per hour

End of Part 1 Preparation Activity

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

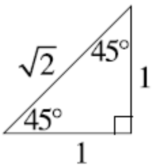
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

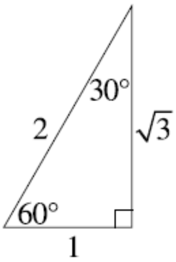
$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

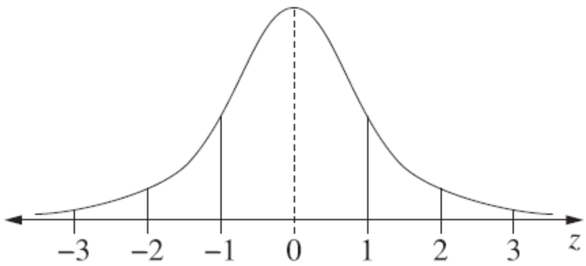
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {^{n}C_{r}} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$