**Name: ____________________**

# Year 12 2023 Extension 2 Mathematics Assessment Task 3

<table>
<tr><td colspan="3"><h2>Investigative Assignment with Validation Task</h2></td></tr>
<tr><td>Task number: 3</td><td>Weighting: 25%</td><td>Due Date: Tuesday 20/6/23 (pds 5 and 6 in Rm 124)</td></tr>
<tr><td colspan="3">Outcomes assessed:<br><br>
MEX12-1 understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts<br>
MEX12-2 chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings<br>
MEX12-3 uses vectors to model and solve problems in two and three dimensions<br>
MEX 12-5 applies techniques of integration to structured and unstructured problems<br>
MEX12-7 applies various mathematical techniques and concepts to model and solve structured, unstructured and multi-step problems<br>
MEX12-8 communicates and justifies abstract ideas and relationships using appropriate language, notation and logical argument</td></tr>
<tr><td colspan="3">Nature and description of the task:<br><br>
As a result of completing this Assignment, students should be familiar with all content related to the following topics:<br>
• 3D Vectors i.e. Chapter 3 of the Extension 2 textbook.<br>
• Further Integration ie. Chapter 6 of the Extension 2 textbook.<br><br>
On the 20th June, 2023 you will receive a selection of similar questions to the Preparation Activity below to complete in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive in the in-class Validation task. NOTE: You will not have to hand in the answers to the questions in this Preparation Activity AND you will not have access to the Preparation Activity during the Validation Task.</td></tr>
<tr><td colspan="3">Non-Completion of Task:<br><br>
If you know you are going to be away on the day of the Validation Task and are unable to hand in / complete both parts of the Assignment on the due day, then you must have supportive documentation. Zero marks will apply if the Assessment Task is submitted/completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.</td></tr>
</table>

**Part 1 Preparation Activity – investigate/attempt each of the following questions in preparation for the in-class Validation Task.**

3D Vectors:

**1** Find the magnitude of $\underset{\sim}{u} = 3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$.

**A** 2

**B** $\sqrt{6}$

**C** $\sqrt{14}$

**D** 14

**2** Find the unit vector in the opposite direction to $\underset{\sim}{u} = 3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$.

**A** $\frac{3}{\sqrt{14}}\underset{\sim}{i} + \frac{1}{\sqrt{14}}\underset{\sim}{j} - \frac{2}{\sqrt{14}}\underset{\sim}{k}$

**B** $\frac{-3}{\sqrt{14}}\underset{\sim}{i} - \frac{1}{\sqrt{14}}\underset{\sim}{j} + \frac{2}{\sqrt{14}}\underset{\sim}{k}$

**C** $\frac{3}{\sqrt{6}}\underset{\sim}{i} + \frac{1}{\sqrt{6}}\underset{\sim}{j} - \frac{2}{\sqrt{6}}\underset{\sim}{k}$

**D** $\frac{-3}{\sqrt{6}}\underset{\sim}{i} - \frac{1}{\sqrt{6}}\underset{\sim}{j} + \frac{2}{\sqrt{6}}\underset{\sim}{k}$

**3** The position vector $\underset{\sim}{u}$ is represented by $\overline{OP}$ and the position vector $\underset{\sim}{v}$ is represented by $\overline{OQ}$.
Which one of the following expressions represents $\overline{PQ}$?

**A** $\underset{\sim}{u} - \underset{\sim}{v}$

**B** $-\underset{\sim}{u} - \underset{\sim}{v}$

**C** $\underset{\sim}{u} + \underset{\sim}{v}$

**D** $\underset{\sim}{v} - \underset{\sim}{u}$

**4** Given that $|\underset{\sim}{u}| = 2$ and $|\underset{\sim}{v}| = 3$ and the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is 45°, what is the scalar product $\underset{\sim}{u} \cdot \underset{\sim}{v}$?

**A** 4.2

**B** 4.5

**C** 5.1

**D** 6.2

**5** What is the scalar product of $\underset{\sim}{u} = 3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} - 2\underset{\sim}{j} + 6\underset{\sim}{k}$?

**A** −5

**B** −1

**C** 7

**D** 11

**6** Which one of these is the equation of the line joining (2, 1, −1) and (0, 3, 1)?

**A** $\underset{\sim}{r} = \left(2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right) + \lambda\left(3\underset{\sim}{j} + \underset{\sim}{k}\right)$

**B** $\underset{\sim}{r} = \left(3\underset{\sim}{j} + \underset{\sim}{k}\right) + \lambda\left(2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right)$

**C** $\underset{\sim}{r} = \left(2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right) + \lambda\left(-2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}\right)$

**D** $\underset{\sim}{r} = \left(-2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}\right) + \lambda\left(-2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}\right)$

**7** A line has equation $\dfrac{x-1}{3} = \dfrac{y-2}{-2} = \dfrac{z+6}{1}$.

Which of the following is a vector parallel to the line?

**A** $3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$

**B** $\underset{\sim}{i} - 2\underset{\sim}{j} + 6\underset{\sim}{k}$

**C** $\frac{1}{3}\underset{\sim}{i} + \underset{\sim}{j} + 6\underset{\sim}{k}$

**D** $-2\underset{\sim}{i} + 4\underset{\sim}{j} + 5\underset{\sim}{k}$

**8** Given the parametric equations of a line, $x = 3 - 5t, y = 2 + t, z = 3 - 2t$, which of the following is the Cartesian equation of the same line?

**A** $\dfrac{x+3}{-5} = \dfrac{y+2}{1} = \dfrac{z+3}{-2}$

**B** $\dfrac{x+5}{3} = \dfrac{y-1}{2} = \dfrac{z+2}{3}$

**C** $\dfrac{x}{-15} = \dfrac{y}{2} = \dfrac{z}{-6}$

**D** $\dfrac{x-3}{-5} = \dfrac{y-2}{1} = \dfrac{z-3}{-2}$

**9** Find the equation of the line through (2, 1, 2) parallel to the line with equation.
$\dfrac{x-2}{3} = \dfrac{y-1}{1} = \dfrac{z-5}{4}$.

**A** $\underset{\sim}{r} = \left(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right) + \lambda\left(2\underset{\sim}{i} + \underset{\sim}{j} + 5\underset{\sim}{k}\right)$

**B** $\underset{\sim}{r} = \left(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right) + \lambda\left(3\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}\right)$

**C** $\underset{\sim}{r} = \left(2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}\right) + \lambda\left(3\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}\right)$

**D** $\underset{\sim}{r} = \left(2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}\right) + \lambda\left(3\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}\right)$

**10** Which of the following lines is parallel to $x = 3 + 2t, y = 2 - t, z = 3 + 4t$?

**A** $\dfrac{x-2}{3} = \dfrac{y-1}{2} = \dfrac{z-5}{3}$

**B** $\dfrac{x-2}{1} = \dfrac{y-1}{3} = \dfrac{z-5}{-1}$

**C** $\dfrac{x-2}{2} = \dfrac{y-1}{-1} = \dfrac{z-5}{4}$

**D** $\dfrac{x-2}{5} = \dfrac{y-1}{1} = \dfrac{z-5}{7}$

**11** How far is the point (3, 4, −5) from the origin?

**12** Find, correct to the nearest degree, the angle between the vectors $2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$.

**13** The vectors $\underset{\sim}{u} = 3\underset{\sim}{i} + n\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} + (2n-1)\underset{\sim}{j} + (4n-2)\underset{\sim}{k}$ are perpendicular.
Find the values of $n$.

**14** Use vectors to prove that the diagonals of a rhombus are perpendicular.

**15** Given 2 non-zero vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ such that $\left|\underset{\sim}{a}+\underset{\sim}{b}\right|=\left|\underset{\sim}{a}-\underset{\sim}{b}\right|$, find $\underset{\sim}{a}\cdot\underset{\sim}{b}$.

**16** Draw a graph represented by the parametric equations $x$ □ 3 sin $t$, $y$ □ 3 cos $t$, $z$ □ $t$, for $0 \le t \le 2\pi$.

**17** Consider the line $M$ with vector equation $\underset{\sim}{m}=\left(3\underset{\sim}{i}+\underset{\sim}{j}+4\underset{\sim}{k}\right)+\lambda\left(2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)$.

Find the coordinates of the point $P$ on the line $\underset{\sim}{m}$ that is nearest to the origin and calculate the distance of the line from the origin.

**18** Find the value of $n$ for which the vectors $\underset{\sim}{a}=-8\underset{\sim}{i}+17\underset{\sim}{j}+12\underset{\sim}{k}$ and $\underset{\sim}{b}=n\underset{\sim}{i}+20\underset{\sim}{j}-13\underset{\sim}{k}$ are perpendicular.

**19** Find the value of $n$ for which the vectors $\underset{\sim}{u}=-24\underset{\sim}{i}-20\underset{\sim}{j}-40\underset{\sim}{k}$ and $\underset{\sim}{v}=6\underset{\sim}{i}+5\underset{\sim}{j}+n\underset{\sim}{k}$ are collinear.

**20** Find the distance of the point (0, 2, 3) from the line $\dfrac{x-3}{2}=\dfrac{y-1}{1}=\dfrac{z+1}{2}$.

**21** The line $L$ goes through $A(4, 3, -2)$ and is parallel to the line $\dfrac{x-1}{3}=\dfrac{y-2}{-4}=\dfrac{z-3}{-1}$.

$P(m, n, -5)$ lies on the line $L$. Determine the values for $m$ and $n$.

**22** Find the perpendicular distance between the point (6, 7, 10) and the line that is parallel to the vector $2\underset{\sim}{i}+\underset{\sim}{j}+\underset{\sim}{k}$ and passes through (5, 9, 4).

**23** Find the perpendicular distance between the lines $\underset{\sim}{p}=\left(\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k}\right)+\lambda_1\left(2\underset{\sim}{i}+\underset{\sim}{j}+3\underset{\sim}{k}\right)$ and $\underset{\sim}{q}=\left(2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)+\lambda_2\left(3\underset{\sim}{i}-\underset{\sim}{j}-\underset{\sim}{k}\right)$.

**24** Consider the line $R$ with vector equation $\underset{\sim}{r}=\left(-2\underset{\sim}{i}+\underset{\sim}{j}+3\underset{\sim}{k}\right)+\lambda\left(-\underset{\sim}{i}-2\underset{\sim}{j}+\underset{\sim}{k}\right)$.

**a** Find the coordinates of the point $P$ on $R$ that is nearest to the point (−2, 5, 8).

**b** What is the distance of the line to the point (−2, 5, 8)?

Further Integration:

**1** Find $\int \frac{1}{9+x^2}dx$ .

**A** $\frac{1}{3}\sin^{-1}\left(\frac{x}{3}\right)+C$ **B** $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

**C** $3\sin^{-1}(3x)+C$ **D** $3\tan^{-1}(3x)+C$

**2** Find real numbers, $A$ and $B$ such that $\frac{4}{x^2-4}=\frac{A}{x-2}+\frac{B}{x+2}$ .

**A** $A=1, B=1$ **B** $A=1, B=-1$

**C** $A=2, B=2$ **D** $A=2, B=1$

**3** If $f'(x)=\frac{x+1}{(x^2+2x+3)^3}$, what is $f(x)$?

**A** $\frac{1}{2(x^2+2x+3)^2}+C$ **B** $\frac{1}{4(x^2+2x+3)^2}+C$

**C** $\frac{-1}{2(x^2+2x+3)^2}+C$ **D** $\frac{-1}{4(x^2+2x+3)^2}+C$

**4** If $\int_3^4 \frac{1}{(x-1)(x-2)}dx = 2\ln M$, what is $M$?

**A** $\frac{\sqrt{3}}{2}$ **B** $\frac{2}{\sqrt{3}}$ **C** $1$ **D** $2\sqrt{3}$

**5** Which integral, with an appropriate substitution, is used to find $\int_1^2 x(2-x)(x^3-3x^2+4)dx$?

**A** $3\int_1^2 u\,du$ **B** $3\int_2^1 u\,du$

**C** $\int_2^0 3u\,du$ **D** $\frac{1}{3}\int_0^2 u\,du$

**6** Using a suitable substitution, which integral can be used to evaluate $\int_1^{e^2} \frac{(\ln x)^2}{x}dx$?

**A** $\int_0^2 \frac{u^2}{e^u}du$ **B** $\int_0^{\ln 2} u^2\,du$

**C** $\int_0^{\ln 2} \frac{u^2}{e^u}du$ **D** $\int_0^2 u^2\,du$

**7** Find $\int e^{\sin x}\cos x\,dx$.

**A** $e^{\sin x} + C$ **B** $e^{\cos x} + C$

**C** $-e^{\sin x} + C$ **D** $-e^{\cos x} + C$

**8** Find $\int \frac{1}{\sqrt{3+2x-x^2}}dx$.

**A** $\frac{1}{2}\sin^{-1}(x-1)+C$ **B** $\frac{1}{2}\sin^{-1}\left(\frac{x-1}{2}\right)+C$

**C** $\sin^{-1}\left(\frac{x-1}{2}\right)+C$ **D** $\sin^{-1}(x-1)+C$

**9** Find $\int x\sin 2x\,dx$.

**A** $-\frac{x}{2}\cos 2x+\frac{1}{4}\sin 2x+C$

**B** $-\frac{x}{2}\cos 2x-\frac{1}{4}\sin 2x+C$

**C** $2x\cos 2x + 4\sin 2x + C$

**D** $2x\cos 2x - 4\sin 2x + C$

**10** Find $\int \frac{1}{x^3+x}dx$.

**A** $\ln\left|x\sqrt{x^2+1}\right|+C$ **B** $\ln|x(x^2+1)|+C$

**C** $\ln\left|\frac{x}{\sqrt{x^2+1}}\right|+C$ **D** $\ln\left|\frac{x}{x^2+1}\right|+C$

**11** **a** Show that $2x^2 + 12x + 18$ can be written as $2(x+3)^2$.

**b** Hence, find $\int \frac{5}{2x^2+12x+18}dx$.

**12** Using $\frac{2x+4}{x^2-4}=\frac{2x}{x^2-4}+\frac{4}{x^2-4}$, show that $\int_3^4 \frac{2x+4}{x^2-4}dx = 2\ln 2$.

**13** **a** Find real numbers $A$ and $B$ such that $\frac{1}{x(x+1)}=\frac{A}{x}+\frac{B}{x+1}$.

**b** Hence, find $\int \frac{1}{x(x+1)}dx$.

**14** Evaluate $\int_2^4 \frac{x^2-2}{x^2+2}dx$.

**15** Use integration by parts to evaluate $\int_1^{2e} \ln x$.

**16** Use integration by parts to find $\int x\sin x\,dx$.

**17** **a** Show that, for the domain $\left(0, \frac{1}{2}\right)$, $\frac{d}{dx}\left(\sin^{-1}\left(\sqrt{2x}\right)\right) = \frac{1}{\sqrt{2x(1-2x)}}$.

**b** Hence find the exact value of $\int_{\frac{1}{8}}^{\frac{1}{4}} \frac{dx}{\sqrt{2x(1-2x)}}$.

**18** Use partial fractions to find $\int \frac{dx}{(x^2+3)(x^2+4)}$.

**19** Let $I_n = \int x^n \ln x \, dx$.

**a** Show that $I_n = \frac{x^{n+1}}{(n+1)^2}\left[(n+1)\ln x - 1\right] + C$.

**b** Hence find $I_3$.

**20** Let $I_n = \int_0^1 \frac{x^n}{1+x^2} dx$ for $n \geq 0$.

**a** Show that $I_n = \frac{1}{n-1} - I_{n-2}$, for $n \geq 2$.

**b** Hence find $I_4$.

**21** **a** Evaluate $\int_0^{\frac{\pi}{4}} \tan^4 x \sec^2 x \, dx$.

**b** Hence, show that $\int_0^{\frac{\pi}{4}} \tan^6 x \, dx = \frac{1}{5} - \int_0^{\frac{\pi}{4}} \tan^4 x \, dx$ and deduce that $\int_0^{\frac{\pi}{4}} \tan^6 x dx = \frac{13}{15} - \frac{\pi}{4}$.

# End of Part 1 Preparation Activity

Mathematics Advanced

Mathematics Extension 1

Mathematics Extension 2

REFERENCE SHEET

## Measurement

### Length

$$l = \frac{\theta}{360} \times 2\pi r$$

### Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

### Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

### Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

## Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

### Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

## Financial Mathematics

$$A = P(1 + r)^n$$

### Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

## Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

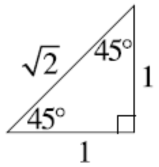
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

## Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$



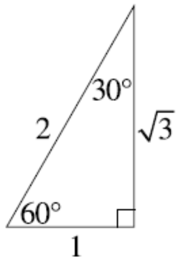

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$




### Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

### Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

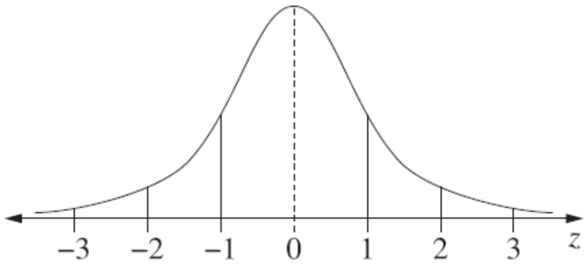
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

## Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

### Normal distribution




- approximately 68% of scores have $z$-scores between –1 and 1
- approximately 95% of scores have $z$-scores between –2 and 2
- approximately 99.7% of scores have $z$-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

### Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

### Continuous random variables

$$P(X \le x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

### Binomial distribution

$$P(X = r) = {}^nC_r p^r(1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x}p^x(1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

## Differential Calculus

| Function | Derivative |
|---|---|
| $y = f(x)^n$ | $\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$ |
| $y = uv$ | $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ |
| $y = g(u)$ where $u = f(x)$ | $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$ |
| $y = \dfrac{u}{v}$ | $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ |
| $y = \sin f(x)$ | $\dfrac{dy}{dx} = f'(x)\cos f(x)$ |
| $y = \cos f(x)$ | $\dfrac{dy}{dx} = -f'(x)\sin f(x)$ |
| $y = \tan f(x)$ | $\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$ |
| $y = e^{f(x)}$ | $\dfrac{dy}{dx} = f'(x)e^{f(x)}$ |
| $y = \ln f(x)$ | $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$ |
| $y = a^{f(x)}$ | $\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$ |
| $y = \log_a f(x)$ | $\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$ |
| $y = \sin^{-1} f(x)$ | $\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$ |
| $y = \cos^{-1} f(x)$ | $\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$ |
| $y = \tan^{-1} f(x)$ | $\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$ |

## Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\Big\{f(a) + f(b) + 2\big[f(x_1) + \cdots + f(x_{n-1})\big]\Big\}$$

where $a = x_0$ and $b = x_n$

## Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

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## Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

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## Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta) = r^ne^{in\theta}$$

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## Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$