Name: ____________________

Year 12 2019 Mathematics Assessment Task 1

Investigative Assignment

Task number: 1	**Weighting:** 20%	**Due Date:** Thursday 6/12/18

Outcomes assessed:

H1 seeks to apply mathematical techniques to problems in a wide range of practical contexts

H2 constructs arguments to prove and justify results

H5 applies appropriate techniques from the study of calculus, geometry, ~~probability,~~ trigonometry ~~and series~~ to solve problems

H7 uses the features of a graph to deduce information about the derivative

H9 communicates using mathematical language, notation, diagrams and graphs

Nature and description of the task:

As a result of completing this Investigative Assignment, students should be able to identify and use linearity properties of the derivative, calculate derivatives of polynomials and solve optimisation problems arising in a variety of contexts involving polynomials on finite interval domains. The ability to choose and use appropriate technology to enhance concept development could also be used in this task.
The Investigative Assignment will consist of two parts:

- Part 1 Preparation Activity (value >> 50% of the overall Investigative Assignment) – completed at home. The suggested time for the Preparation Activity is one week, although you will have approximately 3 weeks to complete it. All answers will need to be completed on your own paper and attached to this booklet. The marks allocated to each question is shown next to the question. All necessary working will need to be shown and answers /responses should be correct / detailed to obtain full marks.
- Part 2 Validation Task (value >> 50% of the overall Investigative Assignment) – to be conducted in class for a period of 50 minutes. The Preparation Activity can be used during the Validation Task and will be handed in together with the Validation Task at the conclusion of the task. Calculators should also be used and all marks for each question will be clearly shown next to each question on the task.

Non-Completion of Task:

If you know you are going to be away on the day of the Validation Task and are unable to hand in / complete both parts of the Investigative Assignment on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is submitted/completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Part 1 Preparation Activity (80 Marks)

Extended investigation – Answer on your own paper and show all working

Problem 1: Which solar panel has the greatest area? (19 Marks)

The area of a glass solar panel needs to be at a maximum to "collect" the greatest amount of sunlight possible. The shape of the flat solar panel is allowed to vary but the metal strip around the edge of the panel (i.e., around the perimeter) is to be kept at a constant value of 8 m to minimise the cost of making the object. For the shapes provided, determine the area of the various panels and hence identify the shape with the maximum area.

A. Glass panel is square (2 marks)

The metal strip is 8 m. Determine the length of each side and hence the area of the square panel.

B. Glass panel is triangular in shape with all sides equal (3 marks)

The metal strip is 8 m. Determine the length of each side and hence calculate the area of the triangular panel using the formula $Area = 0.5ab\sin C$

C. Hexagonal panel (4 marks)

The metal strip is 8 m and the panel is in the shape of a regular hexagon. Draw a labelled diagram to represent the panel identifying the length of the sides and the sizes of the equal angles. Use a dissection method and the formula $Area = 0.5ab\sin C$ to show that the area of the panel is $Area = \frac{3\sqrt{3}}{2}a^2$ where a is the length of each side.

D. Circular panel (3 marks)

The metal strip is 8 m. Determine the radius of the circular panel and hence the area of the panel.

Examining your results

Rank the four shapes provided in order of increasing area. (1 mark)

What feature of these shapes seems to be influencing this order of magnitude? (1 mark)

Suggest a possible range of values for the area of a regular pentagon with a perimeter of 8 m. Justify your choice of values. Calculate the area of the pentagon and relate your answer to the range of values that you predicted. (5 marks)

End of Problem 1

Problem 2: Which raised garden bed can contain the greatest volume of soil? (61 marks)

Raised garden beds come in a variety of shapes and sizes. Four three-dimensional shapes have been suggested for investigation. Assuming each of these shapes can be filled with soil, what is the maximum amount of soil each of the shapes can contain? The dimensions referenced refer to the inner measurements and the thickness of the garden bed can be assumed to be negligible.

For each of the shapes provided

- There is a given restriction on the relationship between some dimensions.
- Identify the formula for the volume of the shape.
- Use Calculus techniques to determine the dimensions that maximise the volume.
- Use the following directions to identify the maximum volume.

A. Garden bed is the shape of a rectangular prism

(a) State the formula for finding the volume of a rectangular prism. (1 mark)

(b) Given $4h+l=5$ metres and the height (h) is half of the width, state the formula for Volume in terms of h only. (2 marks)

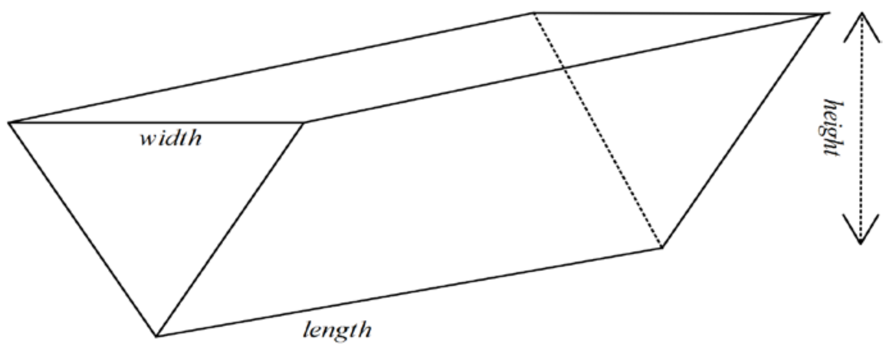
(c) Determine $\frac{dV}{dh}$, the derivative of the expression for Volume. (2 marks)

(d) Let $\frac{dV}{dh}$ = 0 and solve for h. Use calculus to show that the volume is a maximum at that value of h. (4 marks)

(e) Determine the other dimensions of the garden bed. Note that $4h+l=5$ metres and the height (h) is half of the width. (2 marks)

(f) Substitute these values for h, l and w back into the formula for Volume as determined in part (b) and hence determine the maximum volume of soil that can be contained in this garden bed in the shape of a rectangular prism. (2 marks)

B. Garden bed has the shape pictured below



(a) State the formula for finding the volume of this triangular prism. (1 mark)

(b) Given the length plus width is 5 metres ($w+l=5$) and the height (h) is a quarter of the width, state the formula for Volume in terms of w only. (2 marks)

(c) Determine $\frac{dV}{dw}$, the derivative of the expression for Volume. (2 marks)

(d) Let $\frac{dV}{dw}$ = 0 and solve for w. Use calculus to show that the volume is a maximum at that value of w. (4 marks)

(e) Determine the other dimensions of the garden bed. Note that $w+l=5$ metres and the height (h) is a quarter of the width. (2 marks)

(f) Substitute these values for h, l and w back into the formula for Volume as determined in part (b) and hence determine the maximum volume of soil that can be contained in this garden bed in the shape of a triangular prism. (2 marks)

C. Cylindrical garden bed

(a) The relationship between the height and the radius of a cylindrical garden bed is $r+4h=5$ metres. Show that the rule for calculating the volume of the garden bed is $Vol=\frac{\pi}{4}\left(5r^2-r^3\right)$. (3 marks)

(b) Use Calculus techniques to show that the volume is a maximum when the radius is $\frac{10}{3}$ metres. (6 marks)

(c) Determine the height of the garden bed when the volume is maximised. (2 marks)

(d) Determine the maximum volume of this cylindrical garden bed. (2 marks)

D. Garden bed in the shape of a hexagonal prism

The surface of the garden bed is in the shape of a regular hexagon. The volume of the prism is given by $Vol=Area(base)\times h$.

(a) Use the formula for area from Problem 1 plus the restriction that $4h+a=5$ metres, where a is the length of each side of the hexagon, to generate a formula for volume in terms of a only. (3 marks)

(b) Use Calculus techniques to show that a maximum volume of 12.028 m^3 occurs when $a=3.3$ (or $\frac{10}{3}$) metres. (6 marks)

(c) Calculate the height of the garden bed when the volume is maximised. (2 marks)

Examining your results

Enter your results in a table as follows: (3 marks)

Shape	Restriction	Dimensions for maximum volume	Maximum volume
Rectangular prism			
Triangular prism			
Cylinder			
Hexagonal prism			

Comment on your results, considering the following questions.

1. Can you conclude that a particular shape will give a maximum volume? Explain your decision. (2 marks)

2. Are any of the measures i.e., volume or dimensions the same for two or more shapes? (2 marks)

3. Would you expect the length to equal the height in the rectangular prism given this situation? Explain your decision. (2 marks)

4. What other methods, i.e., other than Calculus techniques could be used to determine the maximum volume of these 3-dimensional shapes? What are the advantages of using calculus techniques? (2 marks)

End of Problem 2

End of Preparation Activity