Name: _____

Year 12 2019 Extension 2 Mathematics Assessment Task 1

Assignment with Validation Task

Task number: 1	**Weighting:** 20%	**Due Date:** Tuesday 18/12/18

Outcomes assessed:

- E1 appreciates the creativity, power and usefulness of mathematics to solve a broad range of problems
- E2 chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings
- E3 uses the relationship between algebraic and geometric representations of complex numbers ~~and of conic sections~~
- E9 communicates abstract ideas and relationships using appropriate notation and logical argument

Nature and description of the task:

As a result of completing this Assignment, students should be able to add and subtract Complex Numbers and appreciate that the addition of a Complex Number to another Complex Number corresponds to a translation in the plane. They will be able to multiply Complex Numbers and show that multiplication of a Complex Number by another Complex Number corresponds to a rotation and a scaling of the Complex Number. They will be able to find the conjugate of a Complex Number, divide two Complex Numbers and understand the connection between division and multiplication of Complex Numbers. They will also be able to establish a relationship between algebraic and geometric representations of Complex Numbers. The ability to choose and use appropriate technology to enhance concept development could also be used in this task.
The Assignment will consist of two parts:

- Part 1 Preparation Activity (value >> 50% of the overall Assignment) – completed at home. The suggested time for the Preparation Activity is one week, although you will have 2 weeks to complete it. All answers will need to be completed in the tables provided and on your own paper, attached to this booklet. The marks allocated to each question is shown next to the question. All necessary working will need to be shown and answers/responses should be correct/detailed to obtain full marks.
- Part 2 Validation Task (value >> 50% of the overall Assignment) – to be conducted in class for a period of 50 minutes. The Preparation Activity can be used during the Validation Task and will be handed in together with the Validation Task at the conclusion of the task. Calculators should also be used and all marks for each question will be clearly shown next to each question on the task.

Non-Completion of Task:

If you know you are going to be away on the day of the Validation Task and are unable to hand in / complete both parts of the Assignment on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is submitted/completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Part 1 Preparation Activity

Answer in the tables provided or on your own paper. Show all working

Section A, Student Activity 1: (37 Marks)

Number Systems

Question 1: Complete the following table by writing down two examples of the following (9 marks >> 1 mark each))

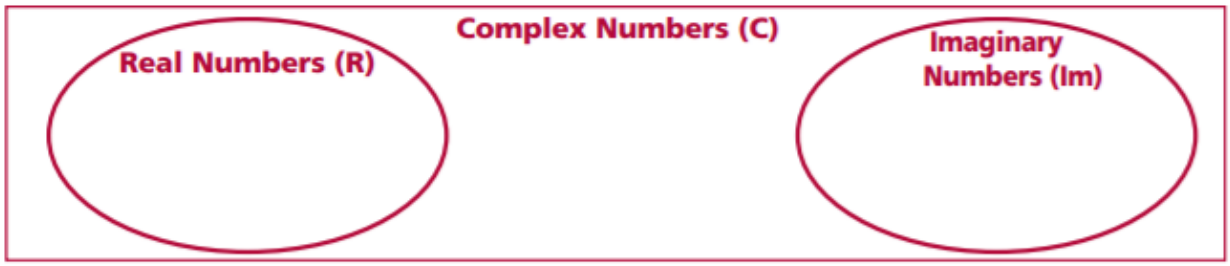
1. Natural numbers	
2. Positive integers	
3. Negative integers	
4. Integers	
5. Rational numbers which are positive and not natural numbers	
6. Rational numbers which are also integers	
7. Irrational numbers	
8. Real numbers	
9. Real numbers which are natural numbers, integers, and rational numbers	

Question 2: Complete the following table by solving the equations and then answering the questions (10 marks >> 2 marks each)

1. Solve $x + 2 = 5$ What type of numbers do you need to solve $x + 2 = 5$?	
2. Solve $x + 7 = 2$ What type of numbers do you need to solve $x + 7 = 2$?	
3. Solve $4x = -3$ What type of numbers do you need to solve $4x = -3$?	
4. $2x + \sqrt{3} = 0$ What type of numbers do you need to solve $2x + \sqrt{3} = 0$?	
5. What types of equations are the above? How many roots / solutions have they?	

Question 3: Can a number be real and imaginary at the same time? Can it be either? Place each of these numbers into the appropriate sets below: Imaginary number set, Real number set, Complex Number set (18 marks >> 3 x 6 marks)

$$\left\{3,\ 0,\ 2+7i,\ 4+0i,\ -5+7i,\ \frac{2}{3}+5i,\ 0+2i,\ i,\ 7-\frac{4}{11}i,\ 5+6i,\ 9,\ 0-\frac{2}{3}i\right\}$$



On graph paper, plot each of the above Complex Numbers on an Argand Diagram. Complete the table below.

Complex Number	Real part	Imaginary part
3		
0		
$2+7i$		
$4+0i$		
$-5+7i$		
$\frac{2}{3}+5i$	$\frac{2}{3}$	5
$0+2i$		
i		
$7-\frac{4}{11}i$		
$5+6i$		
9		
$0-\frac{2}{3}i$		

Section A, Student Activity 2: (8 Marks)

Powers of i (1 mark each)

1. Simplify i^{11} Which answer is correct: ☐ 1 ☐ i ☐ -1 ☐ $-i$	5. Simplify $4i^3 + 7i^9$ Which answer is correct: ☐ $11i$ ☐ $3i$ ☐ $-3i$ ☐ -11
2. Simplify i^{33} Which answer is correct: ☐ 1 ☐ i ☐ -1 ☐ $-i$	6. Simplify $(3i^5)^2$ Which answer is correct: ☐ -9 ☐ $-9i$ ☐ 6 ☐ 9
3. Simplify $i^{16} + i^{10} + i^8 - i^{14}$ Which answer is correct: ☐ 0 ☐ 1 ☐ 2 ☐ i	7. Make up a similar question of your own and explain your answer.
4. Simplify $i^{12} . 3i^2 . 2i^8$ Which answer is correct: ☐ $6i$ ☐ -6 ☐ $-6i$ ☐ 6	8. Make up a similar question of your own and explain your answer.

Section A, Student Activity 3: (18 Marks)

Solving Quadratic Equations (3 marks each)

Quadratic Equation	$ax^2 + bx + c = 0$	Solving using the formula	Roots
$x^2 + 6x + 13 = 0$	$a =$		
	$b =$		
	$c =$		
	$b^2 - 4ac =$		
$x^2 - 4x + 13 = 0$	$a =$		
	$b =$		
	$c =$		
	$b^2 - 4ac =$		
$2x^2 - 2x + 5 = 0$	$a =$		
	$b =$		
	$c=$		
	$b^2 - 4ac =$		

Solving Quadratic Equations continued…

Quadratic Equation	$ax^2 + bx + c = 0$	Solving using the formula (see tables)	Roots
$x^2 - 10x + 34 = 0$	$a =$		
	$b =$		
	$c =$		
	$b^2 - 4ac =$		
$3x^2 - 4x + 10 = 0$	$a =$		
	$b =$		
	$c =$		
	$b^2 - 4ac =$		
$x - \frac{5}{x} = 3$	$a =$		
	$b =$		
	$c =$		
	$b^2 - 4ac =$		

Section A, Student Activity 4: (27 Marks)

The Modulus of a Complex Number (questions 1 to 4 >> 4 marks each, question 5 >> 11 marks)

You will need graph paper with this activity. Use a different Argand Diagram with labelled axes for each question.

1. What is meant by the absolute value or modulus of $z = 5 + 2i$?

   Plot z on an Argand Diagram. Write z as an ordered pair of real numbers:

   Calculate $|z|$

2. Plot $-4i$ on an Argand Diagram. Write $-4i$ as an ordered pair of real numbers.

   Find the distance from (0, 0) to the number $-4i$?

3. Plot as accurately as you can the Complex Number $z = \sqrt{3} + 3i$
   Write this Complex Number as an ordered pair of real numbers.

   Calculate $|z|$

4. Find the modulus of the Complex Number $z = a + i\,b$.

Summarise how you get the modulus or absolute value of a Complex Number by explaining what you do to the real and imaginary parts of the Complex Number.

5. Plot the point $3 + 4i$ on an Argand Diagram. Calculate $|3 + 4i|$
Give the coordinates of 7 other points which are the same distance from the origin.

Plot these points on an Argand Diagram.

What geometric figure contains all the points which are this same distance from the origin?_____

Draw it on the Argand Diagram.

End of Activity A

Section B, Student Activity 1: (12 Marks)

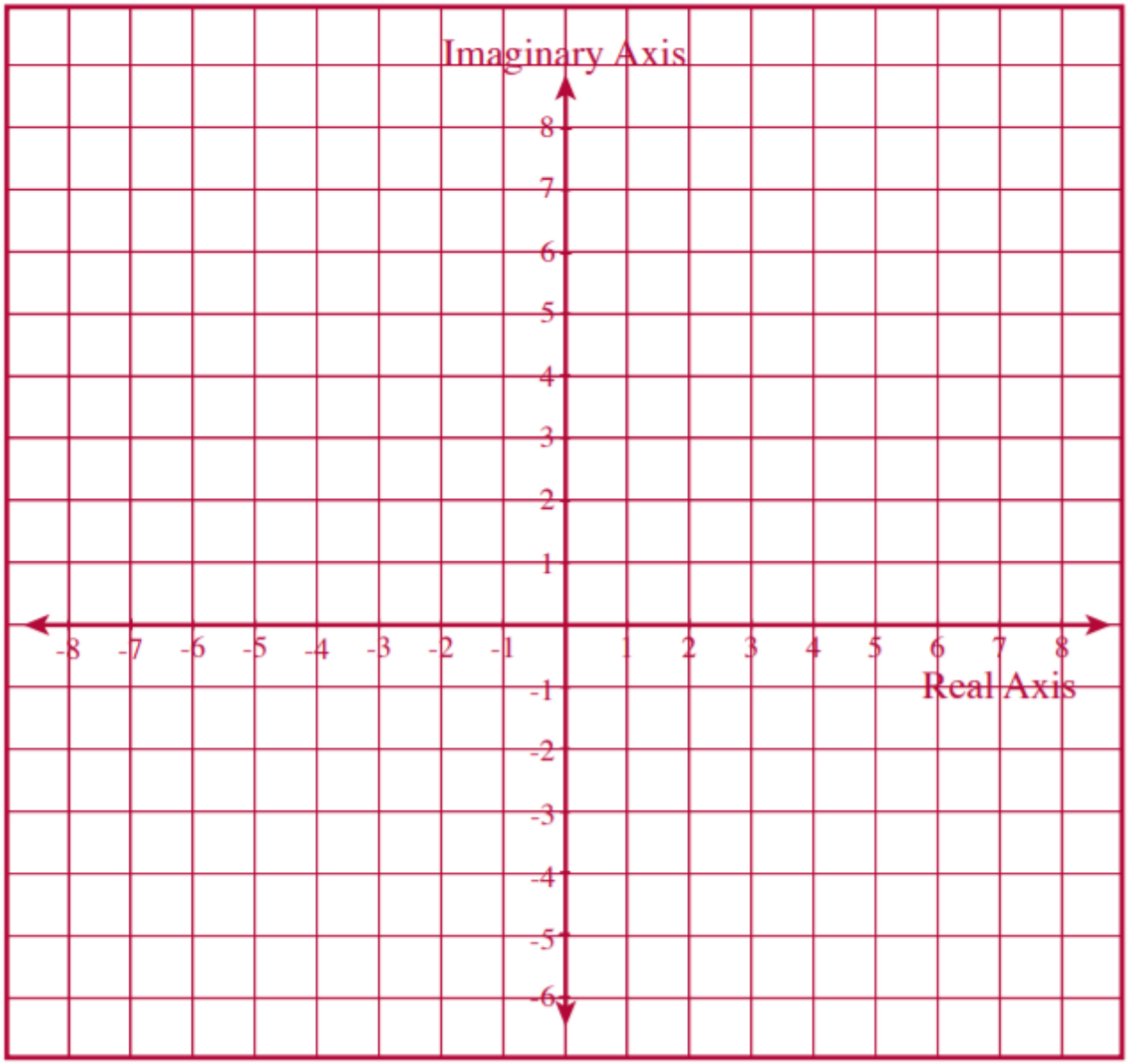
Rule Card (3 marks for each rule)

Addition	Subtaction	Multiplication	Division
Rule # 1	Rule # 2	Rule # 3	Rule # 4
			Example 1:
Example 1:	Example 1:	Example 1:	
Example 2:	Example 2:	Example 2:	Example 2:

Section B, Student Activity 2: (12 Marks)

Addition and Subtraction of Complex Numbers (question 1 >> 4 marks, question 2 >> 8 marks)

1. Add $z = 4 + i$ to each of the following complex numbers:
   o $= 0 + 0i$
   $w_1 = 2 + 2i$
   $w_2 = -3 + 2i$
   $w_3 = 0 + 4i$
2. Represent the complex numberso, w_1, w_2, w_3, as points on an Argand Diagram and then show the results from the above exercise using a directed line (a line with an arrow indicating direction) between each w and its corresponding $w + z$. What do you notice?



End of Activity B

Section C, Student Activity 1: (13 Marks)

Adding and Subtracting Complex Numbers – Practice Questions (questions 1 to 9 >> 1 mark each, question 10 >> 4 marks)

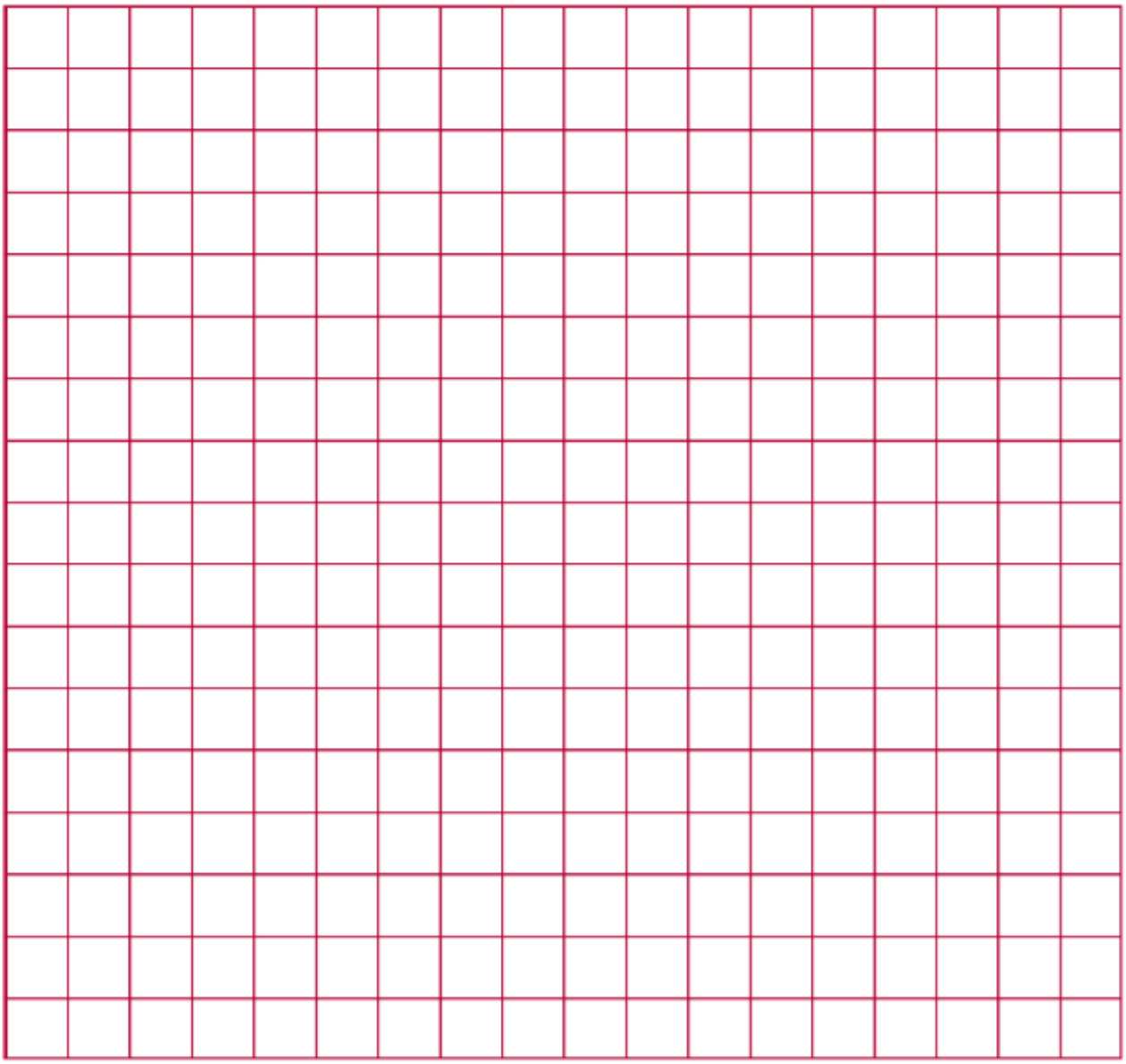
1	$(12 + 4i) + (7 - 11i)$	
2	$(7 - 2i) + (9 - 4i)$	
3	$(4 - 6i) + (-5 - i)$	
4	$(3 - 8i) - (2 - 4i)$	
5	$(-12 - 5i) - (-2 - 8i)$	
6	$\left(2+\frac{1}{3}i\right)+\left(3-\frac{5}{6}i\right)$	
7	$\left(4+\sqrt{-16}\right)+\left(-5-\sqrt{-25}\right)$	
8	$z_1 = 5+i \quad z_2 = -4 + 6i$ $z_3 = -11 + 2i$ Calculate $(z_1 + z_2) - z_3$	
9	$\left(4-\sqrt{-50}\right)-\left(3+\sqrt{-8}\right)$	
10	$z_1 = a + bi, z_2 = c + di$ $z_1 + z_2 =$ $z_2 + z_1 =$ $z_1 - z_2 =$ $z_2 - z_1 =$	

End of Activity C

Section D, Student Activity 1: (17 Marks)

Multiplication by a Real Number (question 1 >> 4 marks, question 2 >> 3 marks, question 3 >> 4 marks, question 4 >> 2 marks, question 5 >> 4 marks)

1. If $z = 3 + 4i$, what is the value of $2z$, $3z$, $5z$, $10z$?
2. Represent the origin $o = 0 + 0i$, z and $2z$ on an Argand diagram.
3. Find the distance z and $2z$ from o, the origin. Describe at least two methods.
4. Comment on your results.
5. Calculate and plot on an Argand Diagram $\frac{z}{2}, \frac{3z}{4}$. What do you notice?

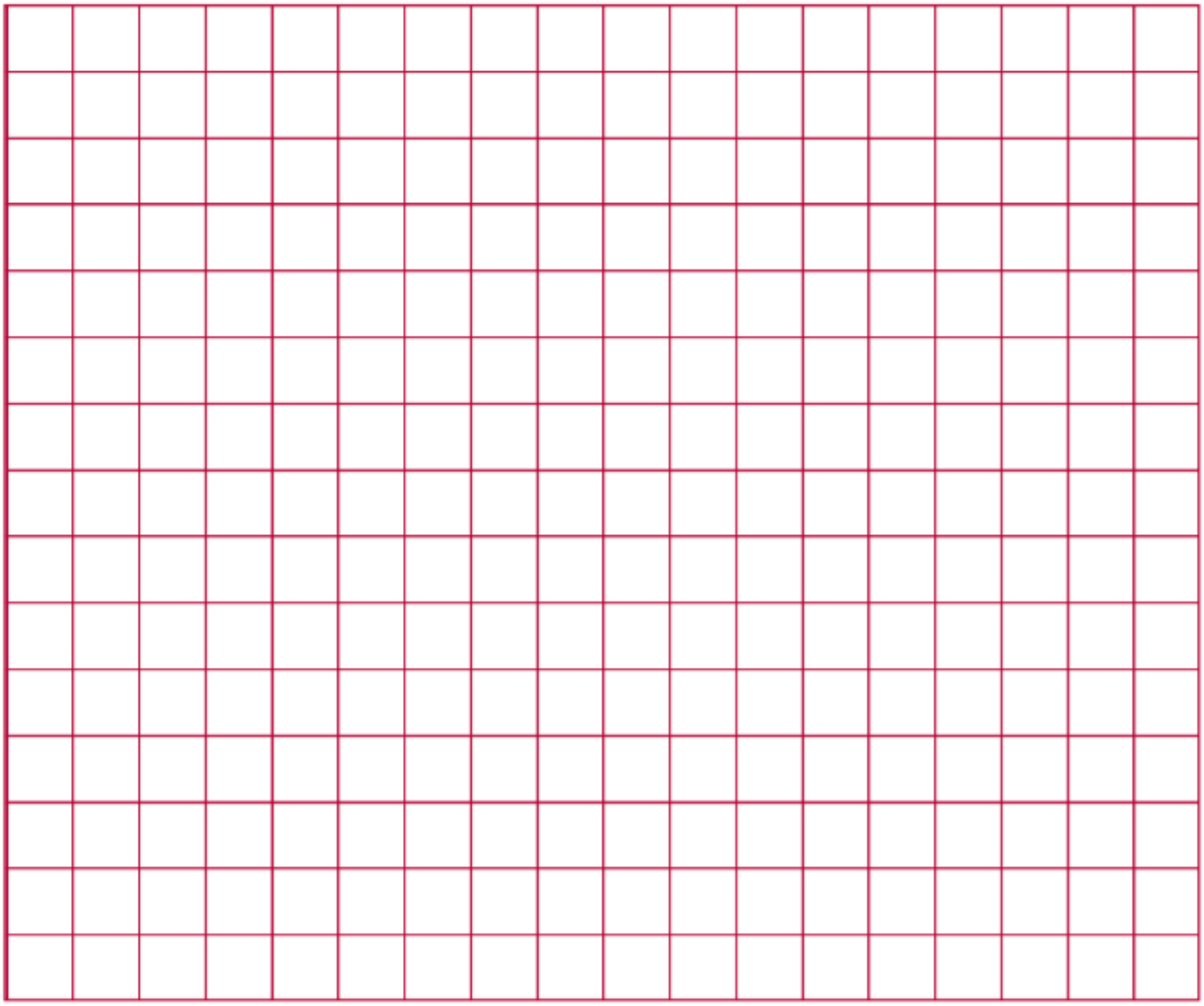


End of Activity D

Section E, Student Activity 1: (19 Marks)

Multiplication by an Imaginary Number (question 1 >> 8 marks, question 2 >> 2 marks, question 3 >> 3 marks, question 4 >> 2 marks, question 5 >> 4 marks)

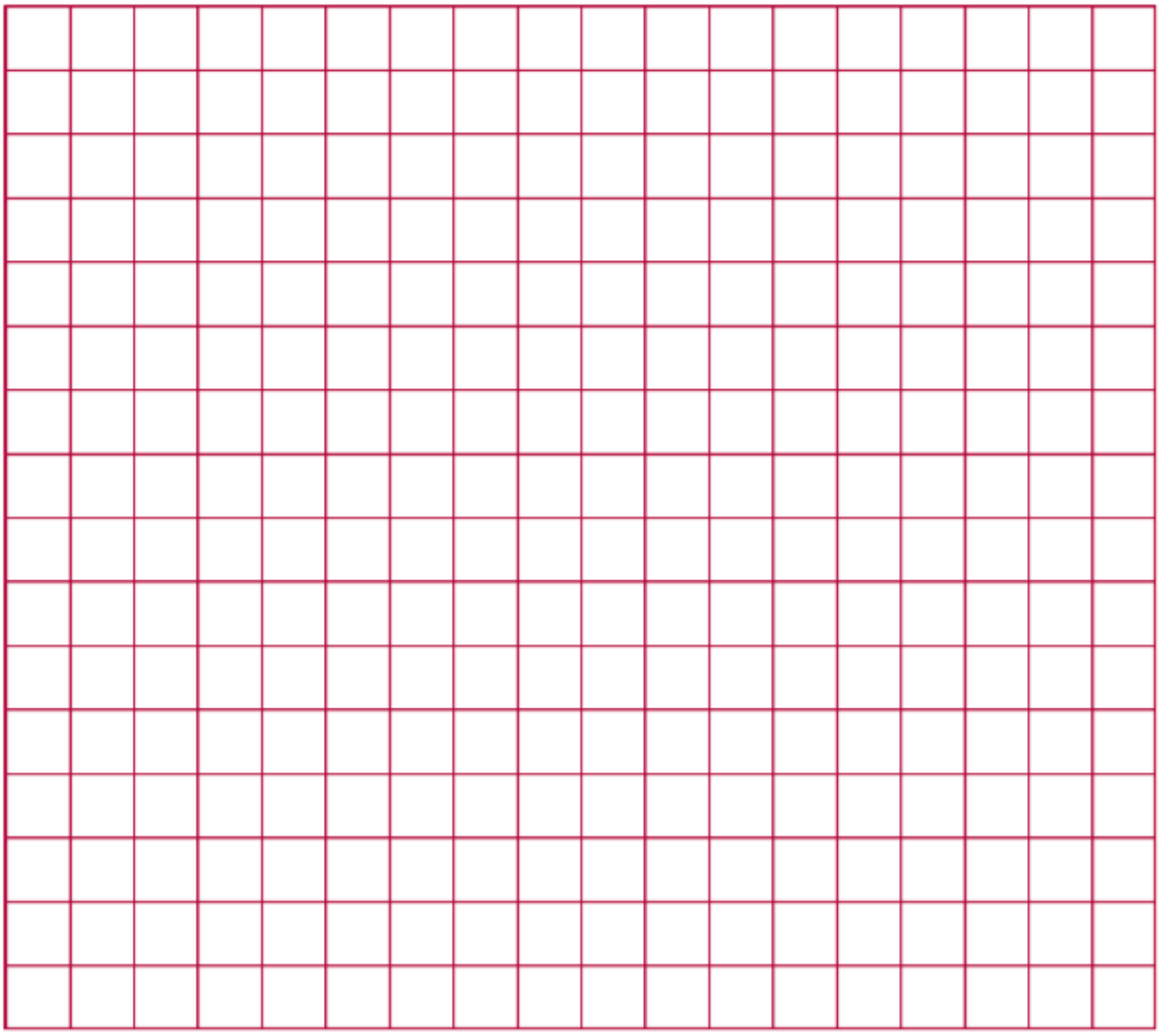
1. If $z = 3 + 4i$, what is the value of iz, i^2z, i^3z, i^4z? Represent your results on an Argand Diagram joining each point to the origin o = 0 + 0i.
2. Investigate what is happening geometrically when z is multiplied by i to get iz? Use geometrical instruments and/or calculation to help you in your investigation.
3. Prove true for the multiplication of iz by i that you get i^2z and the multiplication of i^2z by i that you get i^3z etc.
4. Write your conclusion.
5. Plot on an Argand Diagram $4 + 2i$ and $-i$. Multiply $-i(4 + 2i)$. What do you notice?



Section E, Student Activity 2: (16 Marks)

Multiplication of Complex Numbers in the form $a + i\,b$ (question 1 >> 3 marks, question 2 >> 3 marks, question 3 >> 3 marks, question 4 >> 2 marks, question 5 >> 3 marks, question 6 >> 2 marks)

1. Plot $3 + i$, $1 + 2i$ and their product $1 + 7i$ on an Argand Diagram.
2. Join each point to the origin $o = 0 + 0i$.
3. Measure the angle (by instrument or calculation) made by the line joining $3 + i$ to the origin and the Real Axis and likewise for $1 + 2i$ and $1 + 7i$.
4. What do you notice about the angles?
5. Find the modulus of $3 + i$, $1 + 2i$ and $1 + 7i$.
6. What do you notice?



Section E, Student Activity 3: (49 Marks)

Multiplying Complex Numbers (question 1 >> 7 marks, questions 2 to 3 >> 6 marks each, question 4 >> 9 marks, questions 5 to 6 >> 4 marks each, question 7 >> 7 marks, questions 8 to 10 >> 2 marks each)

When multiplying Complex Numbers all answers are to be given in the form $a+ib$

1	a. Multiply $-4 + 3i$ by 2. b. Plot $-4 + 3i$ and $2(-4 + 3i)$ on an Argand Diagram. c. Calculate $\lvert-4 + 3i\rvert$ and $\lvert2(-4 + 3i)\rvert$. d. What was the effect of multiplication by 2 on $-4 + 3i$?	
2	a. Multiply $-4 + 3i$ by i. b. Plot $-4 + 3i$ and $i(-4 + 3i)$ on an Argand Diagram. c. Calculate $\lvert i(-4 + 3i)\rvert$. d. What was the effect of multiplication by i on $-4 + 5i$?	
3	a. Plot $4 + 2i$ and $-i$ on an Argand Diagram. b. Multiply $-i(4 + 2i)$. c. Plot $-i(4 + 2i)$ on an Argand Diagram. d. What was the effect of multiplication by $-i$ on $4 + 2i$?	
4	a. Plot $1 + i$ on Argand Diagram. b. Calculate $\lvert1 + i\rvert$ c. What angle does the line segment joining $1 + i$ to the origin make with the positive direction of the x axis?	

Multiplying Complex Numbers continued…

4	d. Using what you know about multiplication of one Complex Number by another, what 2 transformations will happen to $1 + i$ if it is multiplied by $(1 + i)$? e. Knowing the modulus of $1 + i$ and the angle it makes with the Real axis, use this information to work out $(1 + i)(1 + i)$. f. Now calculate $(1 + i)(1 + i)$ multiplying them out as you normally would. g. Were you correct in your first answer?	
5	a. Plot $1 + 6i$ and $-1 - 2i$ on an Argand Diagram. b. Multiply $(1 + 6i)(-1 - 2i)$. c. Plot the answer on an Argand Diagram.	
6	If $z_1 = (5 + 4i)$ $z_2 = (3 - i)$ a. Plot z_1 and z_2 on an Argand Diagram. b. Calculate $z_1 . z_2$. c. Plot the answer $z_1 z_2$ on an Argand Diagram.	
7	a. Plot $3 - 2i$ and $3 + 2i$ on an Argand Diagram. What do you notice about both points? b. What angle do you expect the product $(3 - 2i)(3 + 2i)$ to make with the x - axis? Explain. c. Multiply $(3 - 2i)(3 + 2i)$. What do you notice about the answer?	
8	a. Plot $4 + 3i$ on an Argand Diagram b. Plot $(4 + 3i)^2$ on an Argand Diagram	
9	a. Plot on an Argand Diagram: $5 + i(4 - 2i)$	
10	If $z_1 = a + bi$ and $z_2 = c + di$, then a. $z_1 . z_2 =$ b. $z_2 . z_1 =$	

End of Activity E

Section F, Student Activity 1: (29 Marks)

Complex Conjugate (questions 1 to 5, 7 to 11, 13 to 19 >> 1 mark each, questions 6, 12 >> 2 marks each, question 20 >> 8 marks)

For all questions $z_1 = -5 + 4i$ and $w_1 = 3 - 3i$

1	What is $\overline{z}_1$?	
2	What is $z_1 + \overline{z}_1$?	
3	What is $\overline{w}_1$?	
4	What is $w_1 + \overline{w}_1$?	
5	If $z = a + ib$, what is $\overline{z}$?	
6	Calculate $z + \overline{z}$ What type of number is $z + \overline{z}$?	
7	What can you say about 2 Complex Numbers if the sum of the 2 Complex Numbers is real?	
8	What conclusion can you make about the sum of a Complex Number and its conjugate?	
9	Calculate $z_1 - \overline{z}_1$	
10	Calculate $w_1 - \overline{w}_1$	
11	If $z = a + ib$, what is $\overline{z}$?	
12	Calculate $z - \overline{z}$ What type of number is $z - \overline{z}$?	
13	What conclusion can you make about the difference between a Complex Number and its conjugate? i.e. $z - \overline{z}$	
14	Calculate $z_1 . \overline{z}_1$	
15	Calculate $w_1 . \overline{w}_1$	
16	If $z = a + ib$, what is $\overline{z}$?	
17	Calculate $z\,\overline{z}$	
18	What type of number is $z\,\overline{z}$?	
19	When you multiply a Complex Number by its conjugate what type of number do you get?	

Complex Conjugate continued…

20	Remember that when you multiply two Complex Numbers you rotate one of them by the angle the other one makes with the positive direction of the x axis and you stretch the length (modulus) of one by the modulus of the other. Check that this makes sense for $z_1\ \bar{z}_1$ a. Plot z_1 and $\bar{z}_1$ on an Argand Diagram. Join each point back to the origin (o). b. Measure the angle made by oz_1 and the positive direction of the x –axis (θ_1) c. Measure the angle made by o$\bar{z}_1$ and the positive direction of the x –axis. (θ_2) Remember angles measured in an anticlockwise direction from the x –axis are positive and angles measured in clockwise direction from the x –axis are negative d. Rotate oz_1 by θ_2 e. What is $\theta_1 + \theta_2$? f. Multiply $\lvert z_1 \rvert\ \lvert \bar{z}_1 \rvert$ g. Compare the combined transformations of rotating and stretching with $z_1\ \bar{z}_1$.	

End of Activity F

Section G, Student Activity 1: (33 Marks)

Division of Complex Numbers (questions 1 to 7 >> 2 marks each, question 8 >> 1 mark, question 9 >> 14 marks, question 10 >> 4 marks)

Write the following in the form $a + ib$

1.	$\frac{9-6i}{3}$	
2.	$\frac{1}{i}$	
3.	$\frac{7-4i}{1-2i}$	
4.	$\frac{3+i}{3-i}$	
5.	$\frac{2-4i}{-i}$	
6.	$\frac{1}{5-4i}$	
7.	$(5-4i)\left(\frac{5}{41}+\frac{4i}{41}\right)$	
8.	Find the multiplicative inverse of $3 - 2i$	
9.	Calculate the quotient of $\frac{1+7i}{1+2i}$ Plot $1 + 7i$, $1 + 2i$ and their quotient on an Argand Diagram Calculate $\lvert 1 + 7i \rvert$, $\lvert 1 + 2i \rvert$, $\lvert\text{quotient}\rvert$ and investigate if $\frac{\lvert 1+7i \rvert}{\lvert 1+2i \rvert} = \lvert\text{quotient}\rvert$. Calculate the angles that $1 + 7i$ and $1 + 2i$ make with the Real axis and investigate if the subtraction of these angles is equal to the angle that the quotient makes with the Real axis.	
10.	$z_1 = a + bi$, $z_2 = c + di$, Find: $\frac{z_1}{z_2} =$ $\frac{z_2}{z_1} =$	

End of Activity G

End of Preparation Activity