



Name: _____

Mathematics Advanced HSC Assessment Task One 2019 – Assignment

Trigonometric Functions & Discrete Probability Distributions

Task number: 1

Weighting: 20%

Due Date: 6/12/19

Outcomes assessed:

MA 11-4 uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities

MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions

MA11-9 provides reasoning to support conclusions which are appropriate to the context

Nature and description of the task:

As a result of completing this Assignment, students should be familiar with trigonometric functions and their graphs, trigonometric identities and solving trigonometric equations. They should be familiar with random variables, expected values, variance and standard deviations of discrete probability distributions.

On the 6th December, 2019 you will receive a similar selection of the following questions from the Preparation Activity below to complete in 50 minutes in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive on the in-class Validation task. NOTE: You will NOT have access to the Preparation Activity during the Validation Task.

Non-Completion of Task:

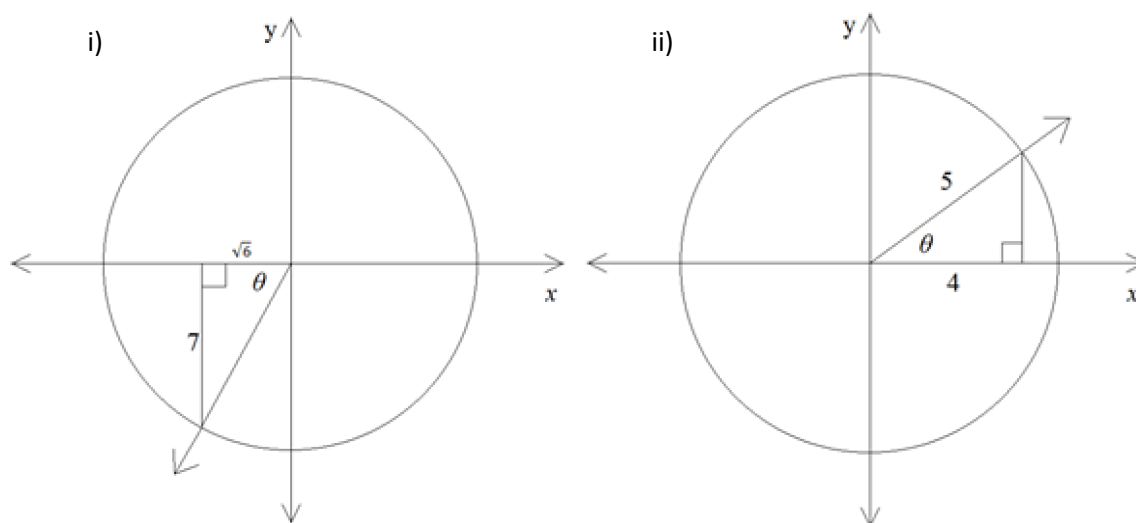
If you know you are going to be away on the day the Assessment Task is due and are unable to hand in Assignment on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is submitted/completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Preparation Activity

PART A: TRIGONOMETRIC FUNCTIONS

QUESTION 1

- a) Use Pythagoras' theorem to find whichever of x , y or r is unknown. Then write down the values of $\sin \theta$, $\cos \theta$ and $\tan \theta$



- b) If $\sin \beta = \frac{\sqrt{7}}{3}$ and β is acute, find the values of $\cos \beta$ and $\tan \beta$.
- c) If $\tan \theta = -\frac{43}{13}$ and $270^\circ < \theta < 360^\circ$, find the values of $\sin \theta$ and $\cos \theta$.
- d) Simplify (all trigonometric functions do not equal zero):
- e) $\frac{1}{\cot \theta}$ ii) $\frac{\sin \theta}{\tan \theta}$ iii) $\operatorname{cosec}^2 \theta - \cot^2 \theta$
- f) Prove the following trigonometric identities:
- i) $3 \sin^2 \theta - 2 = 1 - 3 \cos^2 \theta$ ii) $\sin^2 \theta \cot^2 \theta - \frac{\operatorname{cosec} \theta \tan \theta}{\sec \theta} = -\sin^2 \theta$
- g) Solve each trigonometric equation for $[0^\circ, 360^\circ]$
- i) $\cos x = \frac{1}{\sqrt{2}}$ ii) $\sqrt{3} \sin x - 1 = 0$ iii) $\sin 2x = \frac{1}{2}$ iv) $\cos x = -\sqrt{3} \sin x$
- h) Solve the equation for $0^\circ \leq x \leq 360^\circ$ by reducing it to a quadratic equation in u .

$$2 \sin^2 \theta - 3 \sin \theta = -1$$

QUESTION 2

a) Find the exact value of:

i) $\cos \frac{\pi}{4}$

ii) $\tan \frac{\pi}{6}$

b) Solve for $[-\pi, \pi]$:

i) $\sin x = \frac{\sqrt{3}}{2}$

ii) $\cos x = \frac{1}{2}$

iii) $\tan x = 1$

c) Solve the equation for $[0, 2\pi]$:

$$\cos^2 \theta - \frac{1}{2} \cos \theta = 0$$

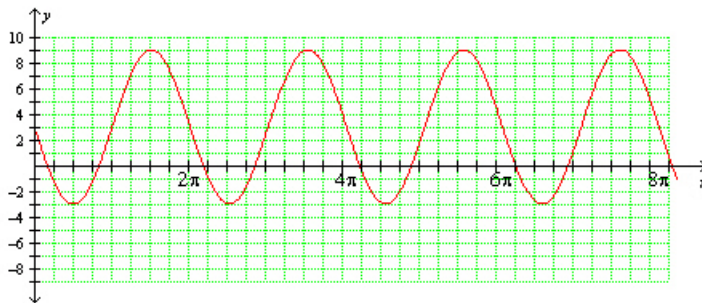
d) i) Which of the six trigonometric graphs have amplitudes, and what are they?

ii) Which of the six trigonometric graphs are periodic, and what are their periods?

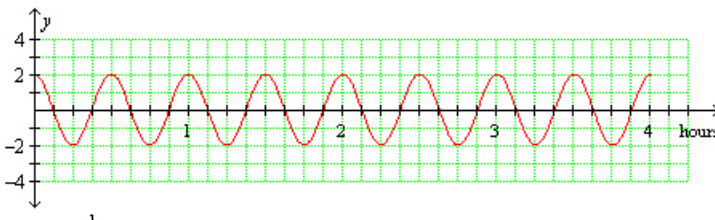
e) i) Give the smallest positive value of θ in terms of π for which $\sin(\theta - x) = \cos x$.

ii) What is the smallest positive value of x in terms of π for which $\sin x = \cos x$?

f) i) What is the amplitude of the trigonometric graph below?



ii) What is the period of the trigonometric graph below?



g) i) Describe the phase shift or horizontal translation of $y = 3\sin(x + \frac{\pi}{8})$. How does it differ from $y = 3\sin x$?

ii) Sketch the graph of $y = 3\sin(x + \frac{\pi}{8})$ for $[-\pi, \pi]$.

h) i) In terms of phase shift how does $y = 2\tan(2x - \frac{\pi}{3})$ differ from $y = 2\tan x$?

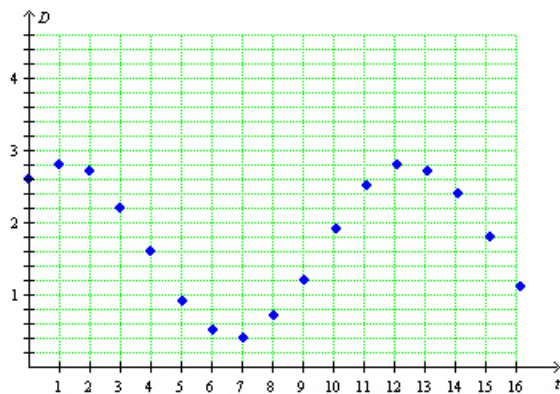
ii) Sketch the graph of $y = 2\tan(2x - \frac{\pi}{3})$ for $[0, 2\pi]$.

i) What is the period of the graph of $y = 4\cos\frac{6x}{5}$?

j) i) Sketch the graphs of $y = 2\sin x$ and $y = \cos 2x$ for $0 \leq x \leq 2\pi$

ii) Hence sketch the graph of $y = 2\sin x + \cos 2x$ for $0 \leq x \leq 2\pi$

k) The graph below shows the depth D m of water over a sandbar, measured every hour using a 24 hour clock, starting at midnight. Use the graph to estimate the depth at 10:30am.



END OF PART A

PART B: DISCRETE PROBABILITY DISTRIBUTIONS

QUESTION 3

a) State whether each random variable is numeric or categorical. If it is numeric, state whether it is discrete or continuous.

- i) The temperature of the coldest day in the Arctic.
- ii) The number of teenagers watching a soccer match.
- iii) The height of a random person that walks into a classroom.
- iv) The religion of a random student at Sydney University.

b) Which of the following are probability distributions

i)

| | | | | |
|------------|---------------|---------------|----------------|----------------|
| x | 1 | 2 | 3 | 4 |
| $P(X = x)$ | $\frac{1}{7}$ | $\frac{3}{7}$ | $\frac{4}{14}$ | $\frac{6}{21}$ |

ii)

| | | | | |
|------------|------|------|------|------|
| x | 1 | 2 | 3 | 4 |
| $P(X = x)$ | 0.15 | 0.45 | 0.25 | 0.15 |

iii)

| | | | | |
|------------|---------------|---------------|----------------|----------------|
| x | 1 | 2 | 3 | 4 |
| $P(X = x)$ | $\frac{1}{6}$ | $\frac{2}{6}$ | $\frac{3}{16}$ | $\frac{8}{24}$ |

c) Give three reasons why the following table is not a valid probability distribution.

| | | | | |
|------------|------|------|------|------|
| x | 1 | 2 | 3 | 4 |
| $P(X = x)$ | -0.1 | 0.25 | 0.62 | 12.4 |

d) Copy and complete each table to find the expected value $E(X)$ of the distribution.

i)

| | | | | | |
|---------|------|------|------|------|-----|
| x | 1 | 2 | 3 | 4 | Sum |
| $p(x)$ | 0.15 | 0.25 | 0.35 | 0.25 | |
| $xp(x)$ | | | | | |

ii)

| | | | | | |
|---------|-----|-----|------|------|-----|
| x | -3 | -1 | 3 | 7 | Sum |
| $p(x)$ | 0.5 | 0.1 | 0.05 | 0.35 | |
| $xp(x)$ | | | | | |

e) When Lochy first visited the Thai Pin Restaurant, he read the menu and assigned each meal a probability indicating how likely he was to order it in the future – this was determined by how much the meal interested him. The fish cost \$22.5 and he rated it $\frac{4}{10}$, the steak cost \$34 and he rated it $\frac{3}{10}$, the vegetarian meal cost \$20 and he rated it $\frac{1}{20}$, the lamb costs \$31 and he rated it $\frac{5}{20}$.

i) What was Lochy's expected cost in buying a meal at the restaurant?

ii) This is now Lochy's favourite restaurant, and he visits it once a week (52 times a year). What is the expected cost to Lochy over the next 2 years, assuming his interest ratings do not change and the prices remain constant?

f) Copy and complete the probability distribution table below to calculate $\text{Var}(X)$ using the definition $\text{Var}(X) = E((X - \mu)^2)$. Also write down σ .

| | | | | | |
|-------------------|------|------|------|------|-----|
| x | 1 | 2 | 3 | 4 | Sum |
| $p(x)$ | 0.15 | 0.20 | 0.35 | 0.30 | |
| $xp(x)$ | | | | | |
| $(x - \mu)^2$ | | | | | |
| $(x - \mu)^2p(x)$ | | | | | |

- g) Now use the alternative formula $Var(X) = E(X^2) - \mu^2$ for the variance for the distribution of the previous question. Copy and complete the table, then calculate the variance.

| | | | | | |
|-----------|------|------|------|------|-----|
| x | 1 | 2 | 3 | 4 | Sum |
| $p(x)$ | 0.22 | 0.43 | 0.17 | 0.18 | |
| $xp(x)$ | | | | | |
| $x^2p(x)$ | | | | | |

- h) Calculate the mean, the variance and the standard deviation of each probability distribution.

i)

| | | | | | |
|--------|------|------|------|------|------|
| x | 0 | 1 | 2 | 3 | 4 |
| $p(x)$ | 0.12 | 0.17 | 0.26 | 0.13 | 0.39 |

ii)

| | | | | | |
|--------|------|------|------|------|-----|
| x | 0 | 1 | 2 | 3 | 4 |
| $p(x)$ | 0.75 | 0.05 | 0.05 | 0.05 | 0.1 |

- i) Explain briefly the meaning and significance of the expected value of a probability distribution.
- j) Explain briefly the meaning and significance of the variance and standard deviation of a probability distribution.
- k) Harris keeps a track of how many times he has to stop for a red light on his way to golf. He records the results in the table below. Calculate the estimated probabilities for stopping at each number of red lights, and hence find the probability he will have to stop at three or more lights.

| Number of red lights (x) | Frequency | Probability $p(X=x)$ |
|------------------------------|-----------|-------------------------|
| 0 | 2 | |
| 1 | 3 | |
| 2 | 9 | |
| 3 | 4 | |
| 4 | 6 | |
| Total | | |