Name: _____

Year 11 Mathematics Extension 1
Preliminary Assessment Task 2

Polynomial Functions

Task number: 2	**Weighting: 30%**	**Due Date: 26/6/20**

Outcomes assessed:

ME 11-1 uses algebraic and graphical concepts in the modelling and solving of problems involving functions
ME 11-2 manipulates algebraic expressions and graphical functions to solve problems
ME 11-7communicates making comprehensive use of mathematical language, notation, diagrams and graphs

Nature and description of the task:

As a result of completing this Assignment, students should be familiar with polynomial functions and their graphs; their roots and coefficients, the division of polynomials, the remainder and factor theorems and solving problems related to polynomials including sketching. They should be familiar with the mathematical language, notation and graphs associated with polynomials.

On the 26th of June, 2020 you will receive a similar selection of the following questions from the Preparation Activity below to complete in 50 minutes in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive on the in-class Validation task. NOTE: You will NOT have access to the Preparation Activity during the Validation Task. You will NOT be given any answers to the Preparation Activity.

Non-Completion of Task:

If you know you are going to be away on the day the Assessment Task is due and are unable to hand in Assignment on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is submitted/completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Note: Due to COVID 19 disruptions, Assessment Task 1 will be postponed for Week 3 Term 3. It will be on Combinations and Permutations.

PREPARATION ACTIVITY

MULTIPLE CHOICE

1. If $P(x) = 3x^4 - 4x^3 - 4x^2 - x - 7$, find $P(3)$.
   A 82
   B 89
   C 311
   D 320

2. If $P(x) = x^3 + 3x^2 + 6x + 6$, divide $P(x)$ by $x + 1$ and hence write $P(x)$ in the form $A(x)Q(x) + R(x)$.
   A $P(x) = (x^2 + 2x + 4)(x + 1) + 2$
   B $P(x) = (x^2 + 4x + 2)(x + 1) + 4$
   C $P(x) = 2(x^2 + 2x + 4)(x + 1)$
   D $P(x) = 4(x^2 + 4x + 2) + (x + 1)$

3. What is the remainder when $4x^4 + 4x^3 + 4x^2 - x - 6$ is divided by $(x + 2)$?
   A 38
   B 44
   C 104
   D 114

4. Which of the following polynomials has a factor of $x - 1$?
   A $3x^3 - x^2 - 3x + 1$
   B $3x^3 - x^2 - 3x + 4$
   C $3x^3 - x^2 - 3x + 8$
   D $3x^3 - x^2 - 3x - 2$

5. What is the value of n if $x^4 - x^3 - 2x^2 + 5x + n$ leaves a remainder of 8 when divided by $(x + 2)$?
   A $n = -4$
   B $n = 2$
   C $n = 8$
   D $n = 14$

6. Write $3x^2 - 2x - 9$ in the form $a(x-1)^2 + b(x-1) + c$.
   A $3(x-1)^2 + 4(x-1) - 8$
   B $3(x-1)^2 - 4(x-1) - 16$
   C $3(x-1)^2 - 5(x-1) + 6$
   D $4(x-1)^2 + 2(x-1) + 6$

MULTIPLE CHOICE (Continued)

7. Find the third factor of $x^3 + ax^2 + bx - 27$ given that $(x - 3)$ is a double factor.
   A $x + 1$
   B $x + 3$
   C $x + 9$
   D $x - 3$

8. Find all the zeros of the polynomial $P(x) = x^3 - x^2 - 30x + 72 = 0$.
   A 3, 4, −6
   B −3, 4, 6
   C −3, −4, 6
   D −1, 4, 18

9. Find the product of the roots of the equation $x^2 - 5x - 3 = 0$.
   A 3
   B −3
   C 5
   D −5

10. If α, β and γ are the roots of $2x^3 + 4x^2 - 5x - 2 = 0$, find $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$.
    A −1
    B −2
    C −5
    D $-\frac{5}{2}$

11. Find the value of a if two of the roots of $x^3 + ax^2 + bx + 10 = 0$ are -5 and 2.
    A 2
    B 1
    C −2
    D −5

12. Find the x-intercepts of the graph of the poynomial $P(x) = (x - 2)(x + 2)(x + 3)(x + 4)$.
    A 2, −2, 3, 4
    B 2, −2, −3, 4
    C 2, −2, 3, −4
    D 2, −2, −3, −4

MULTIPLE CHOICE (Continued)

13. How does the graph of the polynomial $P(x) = -x^5 + 2x^4 + 2x^3 - x^2 + 9x - 6$ behave as $x \to -\infty$ and $x \to \infty$?

A As $x \to -\infty$, $P(x) \to \infty$
As $x \to \infty$, $P(x) \to \infty$

B As $x \to -\infty$, $P(x) \to \infty$
As $x \to \infty$, $P(x) \to -\infty$

C As $x \to -\infty$, $P(x) \to -\infty$
As $x \to \infty$, $P(x) \to \infty$

D As $x \to -\infty$, $P(x) \to -\infty$
As $x \to \infty$, $P(x) \to -\infty$

14. The polynomial $P(x) = (x - 3)^3(x + 1)$ has

A a double root at $x = 3$.
B a triple root at $x = 3$.
C a double root at $x = 1$.
D a triple root at $x = -3$.

15. The polynomial $2x^3$ is divided by $x + 3$. The remainder is:

(A) 54

(B) -54

(C) 216

(D) -216

16. Which one of the following is NOT a polynomial:

(A) $\sqrt{3}x + \sqrt{5}x^2$

(B) $\frac{1}{x^2} - 3$

(C) $\frac{x+1}{5}$

(D) $(x-4)^5$

MULTIPLE CHOICE (Continued)

17. The solution of $x(x-2)^2(3+x)^3 < 0$ is:

(A) $-3 < x < 0$

(B) $x < -3$ or $x > 0$

(C) $x < -3$ or $x > 2$

(D) $-3 < x < 2$

18. If $2x^4 - 3x^2 - x + 4$ is divided by $x^2 - 3x + 1$, the remainder is:

(A) $-58x + 21$

(B) $50x - 15$

(C) $-52x + 23$

(D) $32x - 9$

19. The value of k, if $x + 1$ is a factor of $x^3 - 2x^2 - kx + 5$, is:

(A) 4

(B) 2

(C) -2

(D) -6

20. If $x - 1$ and $x + 3$ are factors of $x^3 - 4x^2 + ax + b$, then:

(A) $a = -15$ and $b = 18$

(B) $a = 15$ and $b = -12$

(C) $a = 21$ and $b = -18$

(D) $a = -15$ and $b = -12$

MULTIPLE CHOICE (Continued)

21. The polynomial $f(x)$ is given by $f(x) = px^3 + 16x^2 + qx - 120$, where p and q are constants. The three zeroes of $f(x)$ are -2, 3 and a. The value of a is:

(A) $\frac{5}{9}$

(B) 4

(C) -5

(D) 5

22. Two roots of the equation $2x^3 + x^2 - px + 6 = 0$ are reciprocals of each another. The value of p is:

(A) 13

(B) 23

(C) -13

(D) -23

23. If α, β and γ are the roots of the equation $2x^3 - 3x^2 + 4x - 5 = 0$, then the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$ is:

(A) $-\frac{3}{5}$

(B) $-\frac{5}{3}$

(C) $\frac{3}{5}$

(D) $\frac{5}{3}$

MULTIPLE CHOICE (Continued)

24. If α, β, γ and δ are the roots of the equation $x^4 - 3x^3 + x - 7 = 0$, then the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}$ is:

(A) $\frac{1}{7}$

(B) $-\frac{1}{7}$

(C) $\frac{3}{7}$

(D) $-\frac{3}{7}$

END OF MULTIPLE CHOICE

PART A

Match each equation to its graph.

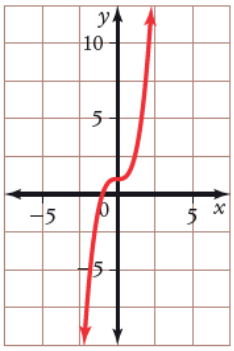
a $f(x) = (x-2)^2 + 1$ **b** $f(x) = x^3 + 1$

c $f(x) = (x-1)^3$ **d** $f(x) = (x+1)(x-2)(x-3)$

e $f(x) = (x-2)^2(x+1)$ **f** $f(x) = (x+2)(x+1)(x-1)(x-3)$

g $f(x) = (x-1)^3(x-3)$ **h** $f(x) = (x+1)^2(x-2)^2$

1



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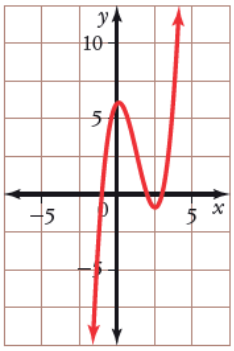

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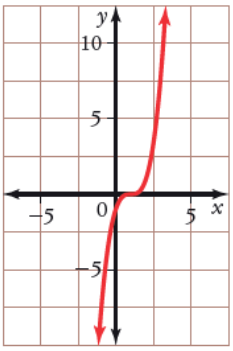

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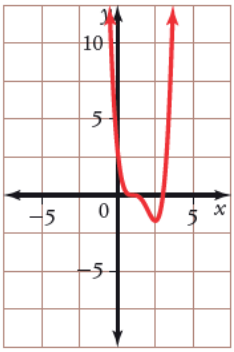

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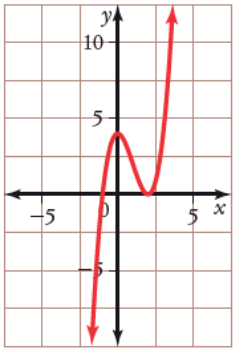

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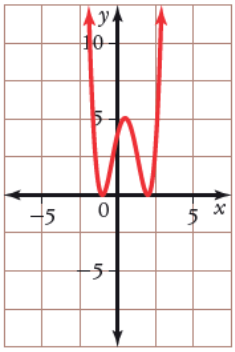

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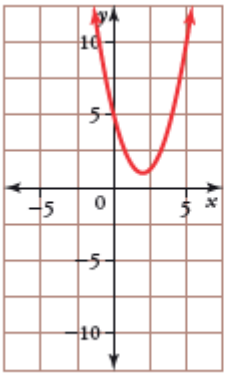

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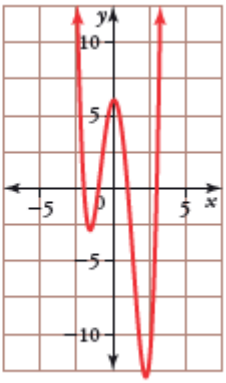

PART B

1 Consider the polynomial $P(x) = 2x^3 - 5x^2 - 6x - 11$. State:

a the degree of $P(x)$,

b the leading coefficient of $P(x)$,

c the leading term of $P(x)$,

d the constant term of $P(x)$,

2 The polynomial $P(x)$ has degree 3. Write down the degree of the polynomial:

a $3P(x)$ **b** $(P(x))^3$

3 Find the coefficient of x^2 in the polynomial $P(x) = (x^2 - 3x - 7)(2x^2 + 4x - 9)$.

4 **a** Sketch the graph of the polynomial function $y = (x + 2)^2(x - 1)(x - 3)$, showing all intercepts with the coordinate axes.

b Hence find the values of x for which $(x + 2)^2(x - 1)(x - 3) < 0$.

5 Sketch the graph of the polynomial $P(x) = x^3 - x^5$.

6 Suppose that the polynomial $P(x) = 2x^3 + 7x^2 - 4x + 5$ is divided by $D(x) = x - 3$.

a Find the quotient $Q(x)$ and the remainder $R(x)$.

b Write down a division identity using the information above.

7 Without long division, find the remainder when $P(x) = x^3 - 5x^2 + 1$ is divided by:

a $x - 3$ **b** $x + 2$

8 **a** Use the factor theorem to show that $x - 2$ is a factor of $P(x) = x^3 - 19x + 30$.

b Hence factor $P(x)$ fully.

9 Find the value of k given that $x + 3$ is a factor of $P(x) = x^3 + 4x^2 + kx - 12$.

10 Find the values of b and c given that $x + 1$ is a factor of $P(x) = x^3 + bx^2 + cx - 7$, and the remainder is -12 when $P(x)$ is divided by $x - 5$.

11 Find the values of h and k given that $x + 2$ is a factor of $Q(x) = (x + h)^2 + k$, and the remainder is 16 when $Q(x)$ is divided by x.

12 The polynomial $P(x)$ is divided by $(x + 1)(x - 2)$. Suppose that the quotient is $Q(x)$ and the remainder is $R(x)$.

a Explain why the general form of $R(x)$ is $ax + b$, where a and b are constants.

b When $P(x)$ is divided by $x + 1$ the remainder is 10, and when $P(x)$ is divided by $x - 2$ the remainder is -8. Find a and b. (Hint: Use the division identity.)

13 Suppose that the polynomial $Q(x) = x^2 - 6x - 4$ has zeroes α and β. Without finding the zeroes, find the value of:

a $\alpha + \beta$ **b** $\alpha\beta$ **c** $\alpha^2\beta + \beta^2\alpha$

d $\frac{1}{\alpha} + \frac{1}{\beta}$ **e** $(\alpha - 3)(\beta - 3)$ **f** $\alpha^2 + \beta^2$

14 If α, β and γ are the roots of the equation $x^3 + 10x^2 + 5x - 20 = 0$, find:

a $\alpha + \beta + \gamma$ **b** $\alpha\beta + \alpha\gamma + \beta\gamma$ **c** $\alpha\beta\gamma$

d $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **e** $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$ **f** $(\alpha + 2)(\beta + 2)(\gamma + 2)$

g $\alpha^2\beta^2\gamma + \alpha^2\gamma^2\beta + \beta^2\gamma^2\alpha$ **h** $\alpha^2 + \beta^2 + \gamma^2$ **i** $\frac{1}{\alpha^2\beta^2} + \frac{1}{\alpha^2\gamma^2} + \frac{1}{\beta^2\gamma^2}$

PART B (Continued)

15 The equation $x^3 + 5x^2 + cx + d = 0$ has roots -3, 7 and α.

- **a** Use the sum of the roots to find α.
- **b** Use the product of the roots to find d.
- **c** Use the sum of the roots in pairs to find c.

16 The equation $6x^3 - 17x^2 - 5x + 6 = 0$ has roots α, β and γ, where $\alpha\beta = -2$.

- **a** Use the product of the roots to find γ.
- **b** Use the sum of the roots to find the other two roots.

17 One root of the equation $ax^2 + 2bx + c = 0$ is the reciprocal of the square of the other root. Show that $a^3 + c^3 + 2abc = 0$.

18 Solve the equation $9x^3 - 27x^2 + 11x + 7 = 0$ given that the roots are $\alpha - \beta$, α and $\alpha + \beta$.

19 Find the zeroes of the polynomial $P(x) = 8x^3 - 14x^2 + 7x - 1$ given that they are $\frac{\alpha}{\beta}$, α and $\alpha\beta$.

20 The polynomial $P(x) = x^3 - x^2 - 16x - 20$ has a double zero.

- **a** Find $P'(x)$ and hence find the double zero.
- **b** Find the remaining zero, and hence factor $P(x)$.

21 The polynomial $P(x) = 3x^4 - 11x^3 + 15x^2 - 9x + 2$ has a triple zero.

- **a** Find the zeroes of $P''(x)$.
- **b** Determine which of the zeroes of $P''(x)$ is the triple zero of $P(x)$.
- **c** Find the remaining zero of $P(x)$.

22 The polynomial $P(x) = x^3 + 3x^2 - 24x + k$ has a double zero.

- **a** Find the two possible values of k.
- **b** For each of the possible values of k, factor $P(x)$.

END OF PREPARATION ACTIVITY