Name: _____

Year 12 2020 Mathematics Extension 2
Assessment Task 3

Assignment – Every Drop Counts

Task number: 3	**Weighting:** 25%	**Due Date:** Friday 7/8/20

Outcomes assessed:

MEX12-1	understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts
MEX12-2	chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings
MEX12-6	uses mechanics to model and solve practical problems
MEX12-7	applies various mathematical techniques and concepts to model and solve structured, unstructured and multi-step problems
MEX12-8	communicates and justifies abstract ideas and relationships using appropriate language, notation and logical argument

Nature and description of the task:

This Assignment provides opportunities for students to develop skills in independent investigation, mathematical modelling and critical thinking. Students will need to apply their knowledge about mechanics and calculus to structured problems as well as their own selected investigation.

Students will be required to communicate and justify the mathematical ideas and relationships using appropriate language, notation and logical argument.

Task overview:

- Three scaffolded learning tasks are to be completed in part A.
- An investigative project involving presentation of work in class is to be completed for part B.

Non-Completion of Task:

If you know you are going to be away on the day of submission/presentation and are unable to hand in/present both parts A and B of the Assignment on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is submitted/completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Part A Learning Tasks (30 Marks)

- Complete the following three questions.
- Show ALL mathematical reasoning, working and diagrams.
- Answer on your own paper and show all working

Question 1: THE TIDE IS HIGH (8 marks)

INQUIRY QUESTION:

When can the Sun Prince cruise ship safely cruise under the Harbour Bridge?

The clearance for shipping under the Harbour Bridge is 45 metres.

The table below shows the tide for Sydney Harbour for Saturday. The first low tide of the day was at 2am and the first high tide was at 8am.

Time	02:00	05:00	08:00	11:00	14:00	17:00
Height	0.4 m	1.2 m	1.5 m	1.2 m	0.4 m	1.2m

Assuming that the tidal motion is simple harmonic motion.

a) Calculate the amplitude and period of the tidal wave. /2

b) The Sun Prince requires 43.8 metres height above the water level to safely cruise under the bridge. This means that the tide will need to be below 1.2metres. The process of sailing under the bridge will take 15minutes. Between what times on Saturday can the ship safely make the passage under the bridge given that no ships are allowed under the Harbour bridge between 8pm and 2am? /6

Question 2: RAIN DROPS (10 marks)

INQUIRY QUESTION:

How fast does a raindrop hit the ground?

Luke was determining how fast his water tank would fill up and found he needed to know how fast the rain would be travelling on average when it entered his water tank. Luke's research found that the average vertical distance covered by a raindrop when it falls to the ground from a rain cloud is given by

$$x = 5\log_e\left(\frac{e^{1.4t}+e^{-1.4t}}{2}\right)$$

where x is in metres and t is the time elapsed in seconds.

a) Show that the velocity of the raindrop, v metres per second, is given by /3

$$v = 7\left(\frac{e^{1.4t}+e^{-1.4t}}{2}\right)$$

b) Hence show that $v^2 = 49\left(1 - e^{-\frac{2x}{5}}\right)$ /3

c) Hence or otherwise, show that $\ddot{x} = 9.8 - 0.2v^2$ /2

d) The physical significance of the constant 9.8 in the acceleration equation is that it represents the acceleration due to gravity. /1

What is the physical significance of the term $-0.2v^2$ in the acceleration equation?

e) Estimate the velocity at which the raindrop hits the ground /1

Question 3: SPRINKLERS (12 marks)

INQUIRY QUESTION:

What is the most efficient water projection to water the vegetables?

When setting up a new sprinkler system at SUNNY VEGETABLE FARM workers Zane and Alfonzo investigated the range and angle of the sprinklers to see where best to place the sprinklers to maximise the water coverage. For their calculations they used gravity as $10ms^{-1}$, air resistance as $\frac{1}{5}mv$. The velocity that the water exits the sprinklers was $12ms^{-1}$ and the sprinklers were at ground height.

a) The initial angle of water projection from Zane's sprinkler was at an angle of 60° above the horizontal. /6

i) Show the horizontal distance is given by $x = 30\left(1 - e^{-\frac{t}{5}}\right)$

ii) Calculate the time it takes the water projection to reach maximum height

iii) Calculate the horizontal range of the water given that the water has covered 75% of the horizontal distance when the projection is at its maximum height.

iv) Calculate the area that the sprinkler covers in one rotation as it rotates back and forwards over an angle of 180°.

b) For Alfonzo's test the test conditions were the same as Zane's, however, the initial angle of water projection was set at an angle of 45° above the horizontal. Calculate the area that Alfonzo's sprinkler covers in one rotation as it rotates back and forwards over an angle of 180°. /4

c) When the crop is planted which sprinkler set up should Sunny Vegetable Farm use? /2

Part B Investigation (15 Marks)

INQUIRY QUESTION

- Design an inquiry question that allows you to investigate a sustainability issue, such as how to reduce water consumption, while demonstrating your understanding of Mechanics and mathematical modelling.
- Possible options could incorporate sprinkler systems, firefighting, street cleaners, fountain designs, shipping logistics … the choice is yours!

RESPONSE

- Demonstrate knowledge, skills and understanding to model and solve complex and interconnected problems in the areas of mechanics and calculus.
- Demonstrate problem-solving and reasoning skills to to model the mechanics of objects with and without resistance and apply these skills to difficult unstructured problems.
- Generate equations that describe the behaviour of an object, analysing and interpreting data values from a real-life situation.
- Use Models of systems to predict the impact of variable changes on the behaviour of objects under the influence of forces such as gravity or air resistance.
- Use mathematical knowledge, skills and understanding to evaluate and justify the inquiry question.
- Supply a bibliography for any resources used.

PRESENTATION

- You will be required to present your investigation the class.
- Use mathematics as an effective means of communication and justification.
- The presentation must include all mathematical reasoning, working and diagrams.
- Communicate in a concise and systematic manner.
- You may choose your format for the presentation.
- As a guide, the presentation should take around 10 minutes.

PART B: INVESTIGATION: MARKING CRITERIA

OUTSTANDING 13-15	▢ Outstanding and insightful independent investigation and evaluation ▢ Presents impressively using mathematics as a means of communication and justification in a complex situation ▢ Demonstrates a highly developed understanding of the behaviour of objects according to mathematical law ▢ Creates a highly sophisticated mathematical model of a physical system and uses extensive knowledge of mechanics to predict the behaviour of objects that are under the influence of forces such as gravity and air resistance. ▢ Clearly and correctly represents mathematical quantities with magnitude and direction and their representation graphically and algebraically. ▢ Extensively evaluates the mathematical model and adequately addresses the inquiry question.
PROFICIENT 10-12	▢ Excellent and well-structured independent investigation and evaluation ▢ Presents effectively using mathematics as a means of communication and justification in a complex situation ▢ Demonstrates a substantial understanding of the behaviour of objects according to mathematical law ▢ Creates a sophisticated mathematical model of a physical system and effectively uses knowledge of mechanics to predict the behaviour of objects that are under the influence of forces such as gravity and air resistance. ▢ Clearly represents mathematical quantities with magnitude and direction and their representation graphically and algebraically. ▢ Effectively evaluates the mathematical model and adequately addresses the inquiry question.
SATISFACTORY 7-9	▢ Satisfactory and reasonably structured independent investigation and evaluation ▢ Presents reasonably using mathematics as a means of communication and justification in a complex situation ▢ Demonstrates a sound understanding of the behaviour of objects according to mathematical law ▢ Creates a satisfactory mathematical model of a physical system and uses knowledge of mechanics to predict the behaviour of objects that are under the influence of forces such as gravity and air resistance. ▢ Satisfactory representation of mathematical quantities with magnitude and direction and their representation graphically and algebraically. ▢ Reasonably evaluates the mathematical model and adequately addresses the inquiry question.
BASIC 4-6	▢ Limited and basically structured independent investigation and evaluation ▢ Presents basically using mathematics as a means of communication and justification in a complex situation ▢ Demonstrates limited understanding of the behaviour of objects according to mathematical law ▢ Creates a basic mathematical model of a physical system and uses limited knowledge of mechanics to predict the behaviour of objects that are under the influence of forces such as gravity and air resistance. ▢ Basic representation of mathematical quantities with magnitude and direction and their representation graphically and algebraically. ▢ Limited evaluation of the mathematical model and addresses the inquiry question in a limited way.
ELEMENTARY 0-3	▢ Elementary and minimally structured independent investigation and evaluation ▢ Elementary presentation using minimal mathematics as a means of communication and justification in a complex situation ▢ Demonstrates elementary understanding of the behaviour of objects according to mathematical law ▢ Creates an elementary mathematical model of a physical system and uses minimal knowledge of mechanics to predict the behaviour of objects that are under the influence of forces such as gravity and air resistance.

	▢ Some representation of mathematical quantities with magnitude and direction and their representation graphically and algebraically. ▢ Limited evaluation of the mathematical model and addresses the inquiry question in a limited way.

End of Assessment Task