Name: _____

Year 12 Mathematics Standard 2
Assessment Task 3

Annuities, Non-linear Relationships and Network Concepts		
Task Number: 3	**Weighting: 25%**	**Due Date: 03/08/20**

Outcomes assessed:

MS2-12-5	makes informed decisions about financial situations, including annuities and loan repayments
MS2-12-6	solves problems by representing the relationships between changing quantities in algebraic and graphical forms
MS2-12-8	solves problems using networks to model decision-making in practical problems
MS2-12-9	chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times and methods for such use
MS1-12-10	uses mathematical argument and reasoning to evaluate conclusions, communicating a position clearly to other

Nature and description of the task:
As a result of completing this Assignment, students should be familiar with:

- Compound interest related problems involving financial decisions. For example, a home loan, a savings account, a car loan or an annuity
- Using an exponential model to solve problems, construct and analyse a quadratic model to solve practical problems. For example, braking distance against speed.
- Reciprocal functions, the rectangular hyperbolic shape of these graphs and their important features.
- Network terminology: vertices, edges, paths, the degree of a vertex, directed networks and weighted edges and solve problems involving network diagrams.

On the 3rd August 2020 you will receive a selection of questions similar to those in this preparation activity booklet. You will have 90 minutes to complete in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive on the in-class Validation Task. Note: You will **NOT** have access to the Preparation Activity during the Validation Task. You will **NOT** be given any answers to the Preparation Activity.

Non-Completion of Task:
If you know you are going to be away on the day the Assessment Task is due and are unable to hand in Assignment on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is submitted/completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

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Preparation Activity

Section I - Multiple Choice

1. A loan is modelled by the recurrence relation $V_{n+1}=V_n \times 1.006 - 350$ where V_n is the balance of the loan after n payments and $V_0=68\,000$. What is the balance of the loan after three payments? Answer correct to the nearest whole number.

   (A) \$68 175

   (B) \$68 467

   (C) \$69 290

   (D) \$70 288

2. The graph $y = 3^x$ has a $y - intercept$ with coordinates:

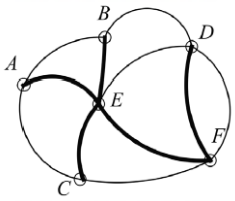
   (A) (0, 3)

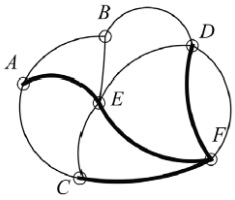
   (B) (3, 0)

   (C) (0, 1)

   (D) (1, 3)

3. Which diagram has a spanning tree for the network shown in bold?

   (A)
   

   (B)
   

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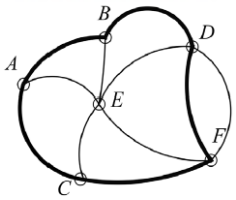
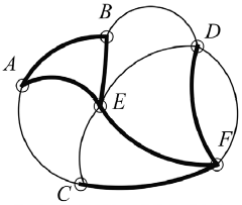
   

   (D)
   

Section I (Continued)

Use the table below to answer questions 4 to 5

Future value of $1				
Period	**1%**	**4%**	**8%**	**12%**
1	1.0000	1.0000	1.0000	1.0000
3	3.0301	3.1216	3.2464	3.3744
5	5.1010	5.4163	5.8666	6.3528

4. Find the future value of $12 000 invested at the end of each year for 5 years at 8% p.a. compounding annually? Answer correct to the nearest whole number.

   (A) $36 000
   (B) $52 896
   (C) $64 996
   (D) $70 399

5. Find the future value of $1400 invested at the end of each month for 5 months at 12% p.a. compounding monthly? Answer correct to the nearest whole number.

   (A) $1400
   (B) $7000
   (C) $7141
   (D) $8894

6. Which of the following points lies on the quadratic curve $y = x^2 - x - 6$?

   (A) (-6, 0)
   (B) (0, -6)
   (C) (2, 4)
   (D) (4, 2)

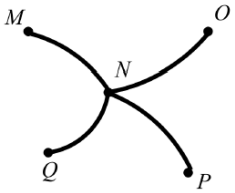
7. What is the maximum value of the quadratic function $y= 2x - x^2$?

   (A) -2
   (B) -1
   (C) 1
   (D) 2

Section I (Continued)

8. Which diagram shows a weighted, directed network?

(A)



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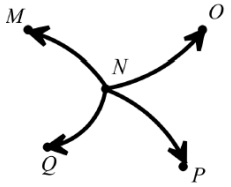
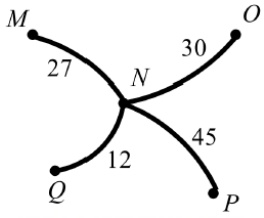
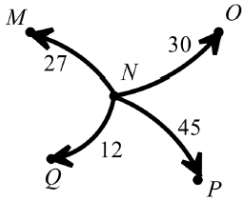


(C)



(D)



Use the table below to answer questions 9 to 10

Present value of $1				
Period	**1%**	**2%**	**4%**	**6%**
2	1.9704	1.9416	1.8861	1.8334
4	3.9020	3.8077	3.6299	3.4651
6	5.7955	5.6014	5.2421	4.9173

9. Find the present value of an annuity where $2,800 is contributed each year for two years into an account earning 6% p.a. compound interest. Answer to the nearest whole number.

(A) $5134

(B) $5768

(C) $13 768

(D) $15 684

Section I (Continued)

10. Find the present value of an annuity where \$1900 is contributed every six months for two years into an account earning 4% p.a. compounding biannually. Answer to the nearest whole number.

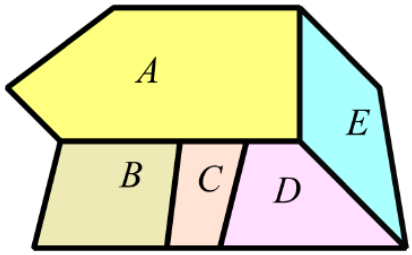
    (A) \$3584

    (B) \$3689

    (C) \$6897

    (D) \$7235

11. The diagram shows a map of five blocks of land which share some boundaries.

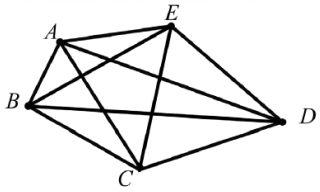


A network is drawn to represent the map.

Vertices indicate blocks of land and edges indicate a shared boundary.

Which network represents the map?

(A)



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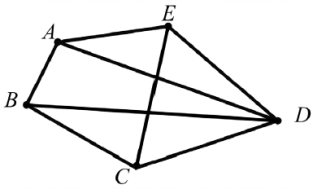

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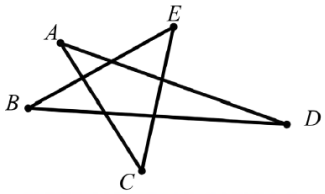

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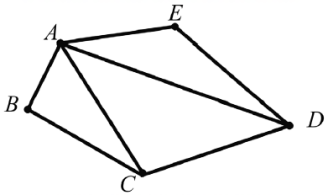
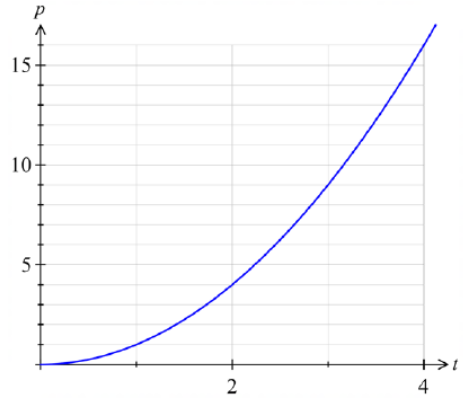


Section I (Continued)

12. The graph opposite shows the population (P) plotted against the time (t) in days.

t	0	1	2	3
p	0	1	4	9



What type of function would model this data?

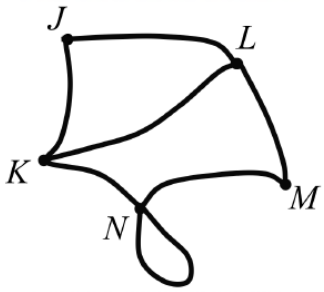
(A) Quadratic

(B) Cubic

(C) Reciprocal

(D) Exponential

13. What is the degree of the vertex N in the network below?



(A) 2

(B) 3

(C) 4

(D) 5

14. The cost per person (c) to use a conference centre is inversely proportional to the number of people attending (n) the conference. If there were 20 people, the cost per person is $60. How many people are required for the cost per person to be $40?

(A) 15

(B) 30

(C) 40

(D) 1200

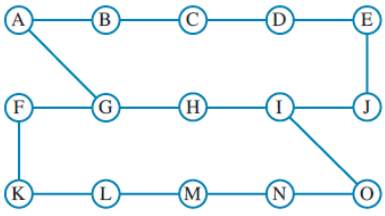
15. Which of the following routes on the network shown could be used to describe a circuit?

(A) AGHIONMLKFGA

(B) IHGFKLMNO

(C) HIJEDCBAGH

(D) FGHIJEDCBAG

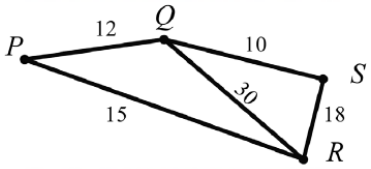


Section I (Continued)

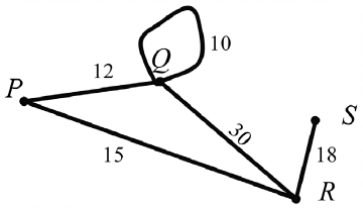
16. Which network is represented by the table below?

	P	Q	R	S
P	-	12	15	-
Q	12	10	30	-
R	15	30	-	18
S	-	-	18	-

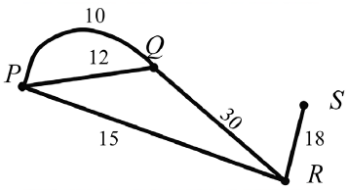
(A)



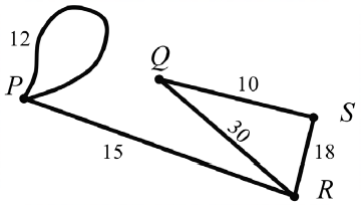
(B)



(C)



(D)



17. The equation $d = 0.4(s^2 + s)$ is used to model the stopping distance for a bicycle where d is the stopping distance in metres and s is the bicycle's speed in m/s. What is the stopping distance given a speed of 5 metres per second?

(A) 5 m

(B) 10 m

(C) 12 m

(D) 15 m

Section I (Continued)

18. Which of the following points lies on the reciprocal function $y = \frac{8}{x}$?

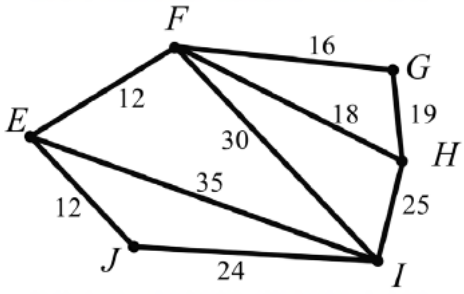
(A) (-2, 8)

(B) (-1, 8)

(C) (0, 8)

(D) (2, 4)

19. Kelly uses Prim's algorithm to find a minimum spanning tree for the network below. She chooses the vertex I as a starting point.



What would be the next vertex added to the spanning tree?

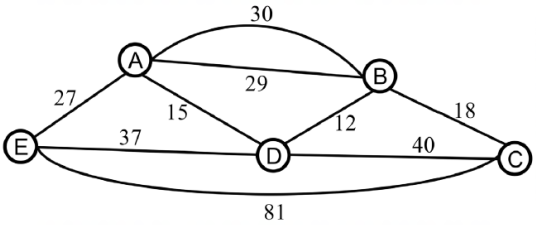
(A) E

(B) F

(C) J

(D) H

20. What is the shortest path from E to C on this network?



(A) EDC

(B) EDBC

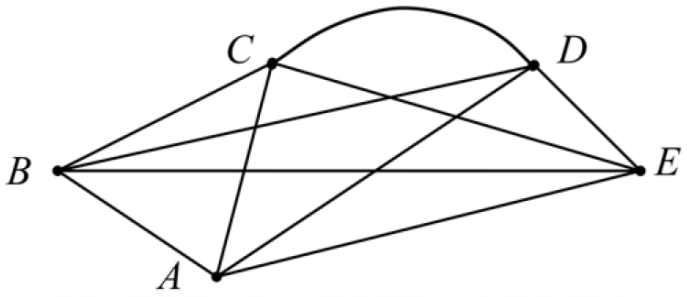
(C) EABC

(D) EADC

END OF SECTION I

Section II

21. A network diagram is shown below.



(A) Find the vertices with an even degree.

(B) Find the vertices with an odd degree.

(C) Find the number of edges.

(D) What is the degree of vertex A?

22. The table below shows the relationships between seven people.

For example, Alex is friends with Charlie, Ethan and Fiona.

Draw a network diagram to represent this table.:

Person	**Friends**
Alex	Charlie, Ethan, Fiona
Danielle	Beatrice, Ethan, Charlie
Fiona	Alex, Beatrice, Gary

23. Let V_n be the balance of a loan after n payments have been made. Write down a recurrence relation model for the balance of a loan of \$60 000 borrowed at 5.4% per annum, compounding monthly, with payments of \$760 per month.

24. Complete the following table of values and graph each function.

(A) $y = 3^x$

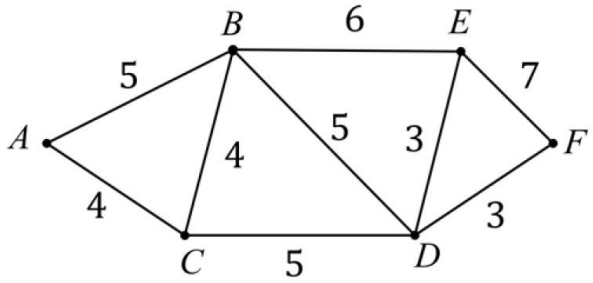
x	−2	−1	0	1	2
y					

(B) $y = \frac{6}{x}$

x	−6	−1	0	1	6
y					

Section II (Continued)

25. What is the length of the shortest path from A to F in the network diagram shown opposite?



26. The table shows the future value of a $1 annuity.

(A) What would be the future value of a $4000 per year annuity at 6% per annum for 12 years, with interest compounding yearly?

(B) What is the value of an annuity that would provide a future value of $52 842 after 6 years at 8% per annum compound interest?

(C) An annuity of $1000 each six months is invested at 12% per annum, compounded biannually for 6 years. What will be the amount of interest earned?

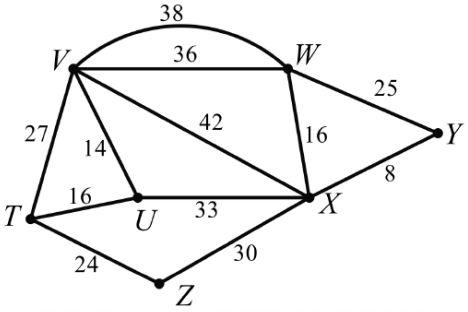
Future value of $1				
Period	**6%**	**8%**	**10%**	**12%**
6	6.98	7.34	7.72	8.12
12	16.87	18.98	21.38	24.13
18	30.91	37.45	45.60	55.75

27. The number of fish increases according to the formula $n = 40(1.5)^t$, where n is the number of fish after t years. How many fish are there after 10 years? (Answer to the nearest whole number).

28. A new water supply network is to be constructed for seven suburbs.

The designers have proposed several possible routes for the pipes which are shown below with their costs in thousands of dollars.

Any of the suburbs can be the source for the network.



Determine the minimum cost of pipes needed to provide water to all suburbs.

Section II (Continued)

29. The table shows the present value of a $1 annuity.

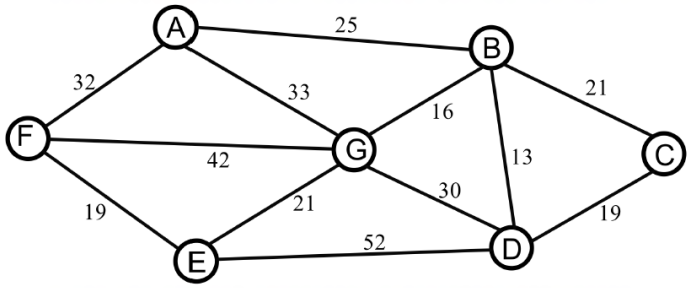
Present value of $1				
Period	**3%**	**4%**	**5%**	**6%**
5	4.5797	4.4518	4.3295	4.2124
6	5.4172	5.2421	5.0757	4.9173
7	6.2303	6.0021	5.7864	5.5824
8	7.0197	6.7327	6.4632	6.2098

(A) What is the present value of a $12 000 per year annuity at 3% per annum for 6 years, with interest compounding yearly?

(B) What is the present value of a $35 000 per year annuity at 5% per annum for 7 years, with interest compounding yearly?

(C) What is the present value of a $2 000 per quarter annuity at 12% per annum for 2 years, with interest compounding quarterly?

30. The network diagram below shows the distances between seven villages in a valley.



(A) Complete the table to represent this network.

	A	B	C	D	E	F	G
A							
B							
C							
D							
E							
F							
G							

(B) Calculate the shortest distance between F and D.

Section II (Continued)

31. The time taken (t) to fit insulation in a school varies inversely with the number (n) of people employed. It takes 6 people 2 days to fit insulation in a school.

    (A) How long does it take 5 people to fit the same insulation in the school?

    (B) How many people are required to fit the insulation in 1 day?

32. Emily wishes to save up enough money so that she could travel the world in the future. She deposits $7525 in a new savings account where she would earn 9% p.a. compounded quarterly.

    (A) What is the quarterly interest rate?

    (B) Emily makes a regular deposit of $1500 every month into the account. What is the recurrence relation of this deposit, where V_n is the future value of the investment after n payments?

    (C) Calculate the balance in her savings account after two years.

    (D) How many months would it take for Emily to save up at least $50 000?

    (E) How many months would it take for Emily to save up at least $50 000 if she had deposited an extra $500 every month?

33. For the functions $y = \left(\frac{1}{3}\right)^x$, $y = 1^x$, $y = 3^x$

    (A) Create a table of values for each function for $-2 \leq x \leq 2$.

    (B) Sketch the graph for all functions on separate number planes.

    (C) For the increasing values of x, determine if the y values are increasing, decreasing or constant for each function.

    (D) Determine if the y values of a function in the form $y = a^x$ where $0 < a < 1$ is always decreasing for increasing values of x.

    (E) Determine if the y values of a function in the form $y = a^x$ where $a = 1$ is always constant for increasing values of x.

    (F) Determine if the y values of a function in the form $y = a^x$ where $a > 1$ is always increasing for increasing values of x.

Section II (Continued)

34. The table below represents a directed network of roads joining four towns Evans, Fuller, Green and Hall.

		To			
		E	**F**	**G**	**H**
From	**E**	-	50	18	-
	F	48	-	30	-
	G	-	33	-	16
	H	-	15	-	15

The weights represent the time of travel in minutes between each town.

(A) Draw a sketch of the network.

(B) Calculate the minimum travel time from Evans to Fuller.

35. A not-for-profit charity organisation provides children in Kenya clothes, clean water, and food.

Use the table below to answer the following questions.

Future value of $1						
Periods	**2%**	**4%**	**6%**	**8%**	**10%**	**12%**
10	10.95	12.01	13.18	14.49	15.94	17.55
20	24.30	29.78	36.79	45.76	57.27	72.05
30	40.57	56.08	79.06	113.28	164.49	241.33
40	60.40	95.03	154.76	259.06	442.59	767.09

(A) For the next 5 years, the organisation raises $75 250 monthly for the children.

If the interest rate is 8% p.a. compounded quarterly, what is the future value of the money that they raised?

(B) They aim to have raised $187 500 000 by the end of 30 years.

If the interest rate was 10% p.a. compounded annually, how much would they have to raise monthly to meet their goal?

Give your answer correct to the nearest dollar.

(C) If the interest rate had increased to 12% p.a. but was compounded biannually. progress

What would be the difference between their goal and the amount that they have raised by the end of the twentieth year?

Section II (Continued)

36. A radioactive element loses 5% of its mass each year.

At the start of 2010, there were 80 mg of the element present.

(A) Prepare a table demonstrating the mass of the radioactive element at the start of each year from 2010 to 2014.

(B) The rule for this decay is $M = I \times (r)^t$, where M is the mass present t years after the start of 2010. Find the values of I and r.

(C) Calculate the decrease in its mass after 10 years. (Round your answer to the nearest whole number)

(D) Sketch the graph of $M = I \times (r)^t$.

37. Using the table below, answer the following questions, to the nearest dollar.

Present value of $1				
Periods	**1%**	**2%**	**3%**	**4%**
10	9.47	8.98	8.53	8.11
20	18.05	16.35	14.88	13.59
40	32.83	27.36	23.11	19.79
60	44.96	34.76	27.68	22.62

(A) Calculate the present value of a 5-year annuity of $70 333 per month at 8% p.a. compounded quarterly.

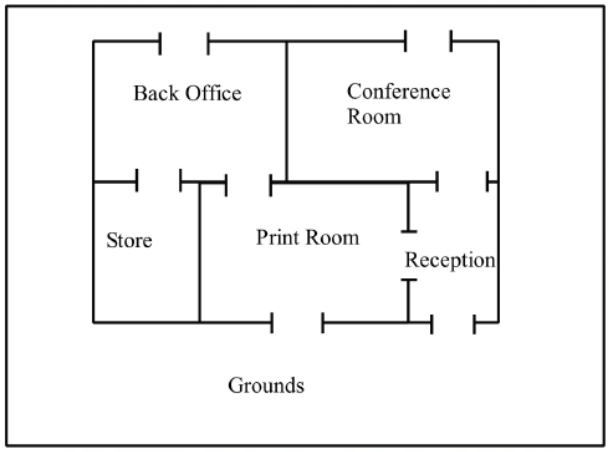
(B) Calculate the monthly contribution of a 10-year annuity whose present value is $382 508 at 12% p.a. compounded quarterly.

(C) Find the yearly interest charged towards a 10-year annuity of $31 805 per half-year compounded annually that has a present value of $542 593.30.

(D) How many quarters does it take for an annuity of $22 242 at 2% p.a. compounded biannually to have a present value of at least $1 000 000?

Section II (Continued)

38. The diagram below shows the plan of a business premises and the grounds on which they stand.



(A) Using the rooms (and the grounds) as vertices, draw a network diagram from the plan.

Use the first letter as the label for each vertex. (e.g. S for the Store, B for Back Office)

(B) On your diagram highlight a spanning tree.

(C) A path from Reception to the Store is RPBS.

List two other paths from Reception to the Store.

39. In an experiment, as a final product of a chemical reaction, a molecule has been found to appear.

If it is known that its mass, M in mg, is increasing by 6% each minute after the start of the reaction (t), answer the below questions by rounding your answer to one decimal place.

(A) Fill the table below.

t(mins)	0	1	2	3	4
M(mg)	60				

(B) Plot the ordered pairs (t, M) on a number plane and join them by sketching an exponential function graph.

(C) Estimate M after 2.5 minutes using the graph you sketched.

(D) Determine the equation of the modelling exponential function.

(E) Find M after 2.5 minutes using the modelling exponential function in (d).

Section II (Continued)

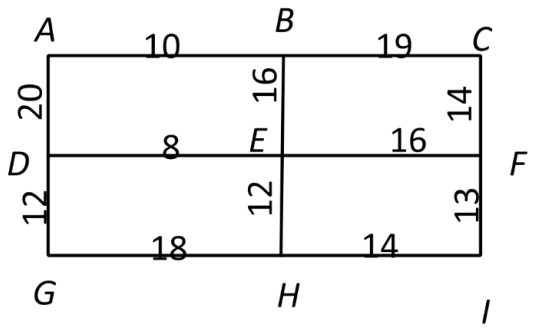
40. Sam has recently gotten engaged and is planning to save $60 000 for his wedding and honeymoon trip in Europe. He currently has $45 600 in his savings account that accumulates interest at 4.8% p.a. compounded monthly and has also decided to save $1 000 each month.

(A) Fill out the following table to summarise the key properties of this annuity and determine how long it would take for Sam to reach his goal.

Payment number	Payment received	Interest earned	Principal increase	Balance of annuity
0				$45 600.00
1 month	$1000.00			
2	$1000.00			
3	$1000.00			
4	$1000.00			
5	$1000.00			
6	$1000.00			
7	$1000.00			
8	$1000.00			

(B) Construct another table like the one above to determine how long would it take to reach Sam's goal if he had begun with $30 000 and saved $1500 each month instead.

41. The network below shows the time (in minutes) to drive along a number of roads.



How long does it take to travel A – D – E – F – I?

Is the path in (a) the fastest way to get from A to I? If not, which path is fastest?

Section II (Continued)

42. Every month you make a payment of $2 000 at 6% p.a. compounded quarterly into your superannuation fund. Your current balance is $250 000.

(A) Construct a table like the one below to summarise the key properties of this annuity and determine the difference between your current balance and your balance at the end of four years.

Payment number	Payment received	Interest earned	Principal increase	Balance of annuity
0				$250 000.00
1 (3 months)				
2 (6 months)				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16 (4 years)				

(B) What percentage of the difference between your current balance and your balance at the end of the first year came from interest earned?

(C) What is the difference in total interest earned in the second and third years?

(D) If the interest rate increased to 8% p.a., still compounded quarterly, how long will it take for the balance to exceeds $350 000?

Section II (Continued)

43. A rectangular swimming pool with perimeter 42 metres is going to be built.

    (A) Represent its length in terms of width.

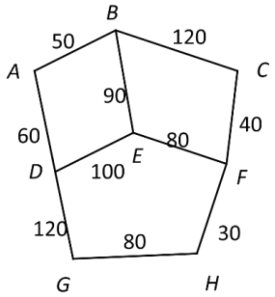
    (B) Represent the base area in terms of width.

    (C) Draw the graph of the area for width ranging from 0 to 21.

    (D) What is the vertex of the graph?

    (E) What can be the maximum base area of the swimming pool?

44. The network below shows a number of houses, and the distances between them in metres.



    (A) What is the shortest distance between A and F?

    (B) What is the shortest distance between C and D?

45. Jake borrows $14 000 to build fishing platforms and shelters around his lake. Interest will be charged at the rate of 9.6% per annum, compounding monthly. Jake will repay the loan with monthly payments of $760.

    (A) What is a recurrence relation model for the balance of this loan?

    (B) Find the value of the loan after 1 month.

    (C) How much interest did Jake pay with the first payment of $760?

    (D) By how much has the balance of the loan been reduced by the first payment of $760?

    (E) Find the value of the loan after five months, correct to the nearest cent.

    (F) Show that Jake paid a total of $507.74 interest, correct to the nearest cent, after five months.

End of Preparation Activity