



Name: \_\_\_\_\_

## Year 12 2020/2021 Mathematics Advanced Assessment Task 1

### Investigative Assignment with Validation Task

**Task number:** 1

**Weighting:** 20%

**Due Date:** Monday  
30/11/20

#### Outcomes assessed:

- MA 11-4 uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities
- MA 11-6 manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems
- MA 11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts
- MA 12-1 uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts
- MA 12-3 applies calculus techniques to model and solve problems
- MA12-10 constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

#### Nature and description of the task:

As a result of completing this Investigative Assignment, students should be familiar with the logarithmic and index laws, functions involving logarithms and exponentials including the derivative of the natural exponential function. They will also be able to answer simple questions about probability, use a variety of diagrams to represent multi-stage events and answer questions of the same including conditional probability. They will be able to solve trigonometric equations, use the trigonometric identities and graph trigonometric functions.

On the 27<sup>th</sup> November, 2020 you will receive a selection of similar questions to the Preparation Activity below to complete in 50 minutes in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive in the in-class Validation task. NOTE: You will not have to hand in the answers to the questions in this Preparation Activity AND you will not have access to the Preparation Activity during the Validation Task.

#### Non-Completion of Task:

If you know you are going to be away on the day of the Validation Task and are unable to complete it on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

**Part 1 Preparation Activity – investigate/attempt each of the following questions in preparation for the in-class Validation Task.**

**Probability:**

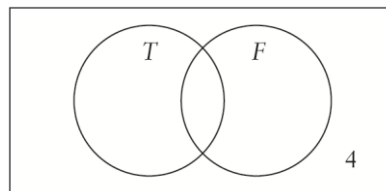
- 1  $A = \{2, 3, 5, 7, 11\}$ ,  $B = \{1, 3, 5, 7, 9\}$ . Find  $A \cap B$ .
- 2 Name an event has the same probability as tossing a tail on a coin?
- 3 This table displays the number of cars owned by each home in Grove Ave.

Number of cars	Number of homes
0	1
1	5
2	9
3	5
4	3

Find the relative frequency that a home has 3 cars.

- 4 Elizabeth buys 5 tickets and Tania buys 6 tickets in a raffle where a total of 300 tickets were sold. Only one prize is to be won. Write an expression that will calculate the probability that neither Elizabeth nor Tania win the raffle prize?
- 5 The probability of a train on a city line running late is estimated at 12%. If there are 225 trains running on a city line each day, how many would be expected to arrive on time?
- 6 Which is the correct formula for  $P(A|B)$ ?  
**A**  $\frac{P(A \cap B)}{P(A)}$       **B**  $\frac{P(A \cap B)}{P(B)}$   
**C**  $\frac{P(A \cup B)}{P(A)}$       **D**  $\frac{P(A \cup B)}{P(B)}$
- 7 From a group of 30 Year 11 students, 22 students study Business Studies, 9 students study Japanese (J) and 4 study neither. What is the probability that one student chosen at random from this group studies Business Studies but not Japanese?
- 8 A horse has a probability of 48% of winning its first race and a 36% chance of winning both its first and second race. What is the probability of the horse winning the second race, given that it wins the first race?

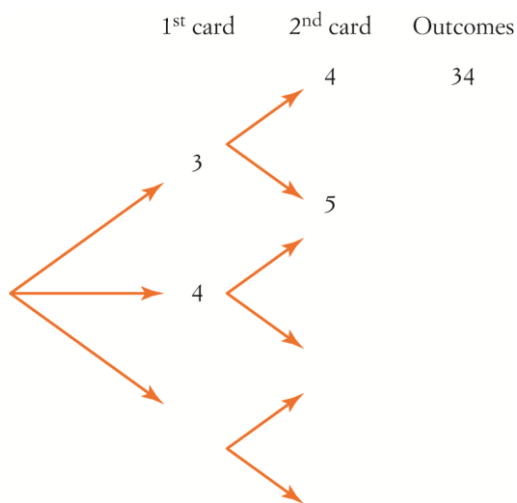
- 9 A box of chocolates has an equal number of dark and white chocolates. Two chocolates are chosen at random. What is the chance that at least one is a white chocolate?
- 10 For a double-scoop ice cream cone, Michelle will choose 2 different flavours at random from chocolate, vanilla, strawberry and mango. What is the chance that Michelle chooses chocolate and mango?
- 11 The 11 letters of the word MATHEMATICS are written on separate cards and placed in a bag. One card is chosen at random. What is the probability of choosing a card with the letter
- M?
  - M or A or T?
- 12 A pair of dice are rolled. What is the chance that one die shows the number 2 and the other shows a number greater than 3?
- 13 In a survey of 29 students, 6 are in training for both athletics track (T) and field (F) events while 14 altogether do track only and 4 students do not do any track or field events.
- Complete this Venn diagram.



- What is the probability that a randomly-selected student does track events?
- What is the probability that, if 2 students are randomly selected, neither do any athletic events?

14 The numbers 3, 4 and 5 are written on separate cards and placed in a bag. Two cards are drawn at random, one after the other without replacement, to form a two-digit number.

a Complete this tree diagram.

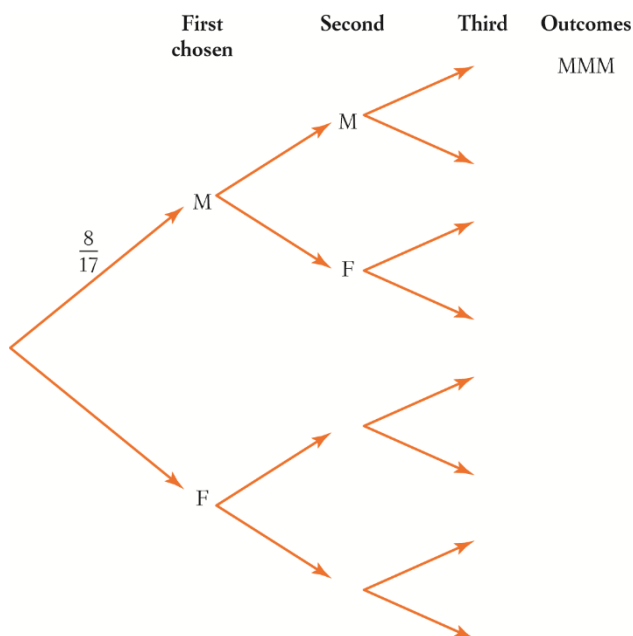


b What is the probability of forming a 2-digit number that is:

- i an odd number?
- ii greater than 40?
- iii a multiple of 5?

15 In a class there are 8 male students and 9 female students. Three students are chosen at random.

a Complete the probability tree, showing the respective probabilities on its branches and list all the possible outcomes.



b What is the probability that for the 3 students:

- i all are females?
- ii at least one is male?

- 16** In a teacher's pencil case there are 3 black, 5 red and 2 green pens.
- If a pen is chosen at random, what is the probability that it is a black pen?
  - If 2 pens are chosen at random, what is the probability that both pens chosen are red if:
    - the first pen is replaced before the second is taken?
    - both pens are taken out together?
- 17** Roger has a 70% chance that he will win his tennis match, while Serena has a  $\frac{4}{5}$  chance of winning her match. What is the probability that:
- Roger wins, but Serena doesn't win?
  - at least one of them wins their match?
- 18** A golf bag contains 6 white, 2 yellow and 4 orange golf balls. Four balls are taken without replacement. What is the probability of drawing:
- all orange balls?
  - 2 white and 2 orange balls?

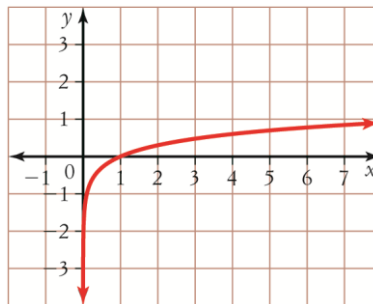
- 19** Students at Westvale High School chose their summer sport.

Sport	Male	Female
Swimming	82	110
Cricket	61	13
Basketball	119	91
Squash	44	14
Touch football	58	65
Dance	4	33

- What is the ratio of male to female students who participated in this survey?
  - What is the relative frequency of a student chosen at random playing touch football?
  - What is the relative frequency of a student chosen at random playing touch football or basketball?
  - What is the relative frequency of a male student chosen at random playing basketball?
- 20** A football team has a probability of 70% of winning when Ronaldo is playing in the team. When Ronaldo is not playing, the team has a probability of 40% of winning. The probability of the team having Ronaldo playing at any one time is 90%.
- Find, as a percentage correct to 2 decimal places, the probability that Ronaldo was playing when the football team won a match.

## Exponential and Logarithmic Functions:

- 1 Evaluate  $e^5$  to 2 decimal places.
- 2 Differentiate  $y = (e^x + 7x)^3$ .
- 3 Find the gradient of the tangent to the graph of  $y = e^x + 2$  at  $x = -1$ .
- 4 Evaluate  $7^{\log_3 9}$
- 5 Solve the equation  $3 \ln 2 + \ln 5 - \ln 10 = \ln x$ .
- 6 Which function below has this graph?



- A**  $y = \log x$                       **B**  $y = e^x$   
**C**  $y = ba^x$                         **D**  $y = \log |x + 1|$
- 7 Which statement is true if  $5^x = 35$ ?  
**A**  $x = \log_{10} 35$                       **B**  $x = \frac{\log_{10} 35}{\log_{10} 5}$   
**C**  $x = \frac{\log_{10} 5}{\log_{10} 35}$                       **D**  $x = \frac{\log_{10} 35}{5}$
  - 8 Evaluate  $\log_7 37$ .
  - 9 If  $\log_e 5 = 1.6$  and  $\log_e 9 = 2.2$ , then which is the correct expression for  $\log_e 45$ ?  
**A**  $1.6 \times 2.2$                       **B**  $1.6 \div 2.2$   
**C**  $1.6 - 2.2$                         **D**  $1.6 + 2.2$
  - 10 Solve the equation  $21 = 8e^{0.024t}$ .

**11** Graph each function.

**a**  $y = 3^x + 2$

**b**  $y = 3^{x-1}$

**c**  $y = \log(x + 1)$

**d**  $y = \log x - 2$

**12 a** Find the exact value of the gradient of the normal to the graph of  $y = e^x + 5x^2$  at  $x = 1$ .

**b** Hence find the equation of the normal to the graph at  $x = 1$ .

**13** Simplify each expression.

**a**  $2 \log_{10} 5 + \log_{10} 4 - \log_{10} 2$

**b**  $\log_3 9xy^2 - \log_3 27xy$

**c**  $\log_4 16x^4 - \log_4 (2x)^2$

**d**  $\ln\left(\frac{e^x x^2}{y}\right) + \ln e^x y$

**14** Solve each equation.

**a**  $\ln(x + 1) = 1.72$

**b**  $e^{2x} = 6.27$

**c**  $\log_5 x = 3$

**d**  $15 = 12e^{x-2}$

**e**  $11^{5x-2} = 5$

**f**  $\log_x 5 = \frac{1}{5}$

- 15** The number of bacteria in a sample of water from an infested lake is increasing by 5% per hour. Initially, there were 1500 bacteria when testing started.
- Write an equation that represents the number of bacteria after  $t$  hours.
  - How many bacteria are present after 2 days?
  - A pesticide was introduced 10 hours after the initial count. The pesticide had a positive result in controlling the growth of bacteria, and reduced it by 7% per hour. After how many hours will the number of bacteria be reduced to 500?
- 16** Evaluate each expression correct to 3 significant figures, where possible.
- $\log_{10} 50 - \log_{10} 5$
  - $e^{\ln 5} \div e^{\ln 6}$
  - $e^2 + e^{-3}$
  - $\frac{2e^3}{\log_{10} 4}$
- 17** State the domain, range and asymptotes of each function.
- $y = 2 \log_e x$
  - $y = 2 \times 10^x$
- 18**
- Write  $\log_3 729 = 6$  in index form.
  - Write  $3^{-3} = \frac{1}{27}$  in logarithm form.
- 19** The population of a Pacific island is given by  $P = P_0 e^{kt}$ , where  $P_0$  is the initial population,  $t$  is the time in years after the year 2004 and  $k$  is a constant. In 2004, the population was 10 200. In 2009 (5 years later), the population increased to 17 250.
- What is the value of  $P_0$ ?
  - What is the value of  $k$ ?
  - What will be the population in 2020?
  - How many years will it take for the population to reach 100 000?
  - If the area of the island is 1525 km<sup>2</sup>, how many years will it take for the island to be overpopulated at a population density of 100 persons per 1000 m<sup>2</sup>?



## Trigonometric Functions:

- 1 Convert  $\frac{3\pi}{7}$  to degrees.
- 2 Find the exact value of  $\sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)$ .
- 3 Find the exact value of  $\cos(225^\circ)$ .
- 4 Find the value of  $t$  if  $\tan(2t + 15) = \cot(6t - 5)$ .
- 5 Compared to  $y = \cos x$ , the function  $y = 3 \cos 2x$  has:
  - A the same amplitude and period
  - B a larger amplitude and a shorter period
  - C a smaller amplitude and a shorter period
  - D a smaller amplitude and a larger period
- 6 A circle has a radius of 25 cm. An arc on the circle measuring 37 cm subtends an angle  $\theta$  at the centre of the circle. Calculate  $\theta$  in radians.
- 7 Find the exact value of  $\sin(-300^\circ)$ .
- 8 Given that  $\sin \theta = \frac{12}{13}$ , find the exact value of  $\cos \theta$ .
- 9 Which statement is true?
  - A  $\sin\left(x + \frac{\pi}{2}\right) = -\cos x$
  - B  $\cos\left(x - \frac{\pi}{2}\right) = \sin x$
  - C  $\sin\left(x - \frac{\pi}{2}\right) = \cos x$
  - D  $\cos\left(x + \frac{\pi}{2}\right) = \sin 2x$
- 10 Solve  $2\cos x = \sqrt{3}$  for  $0 \leq x \leq 2\pi$ .

11 If  $\sin \theta = -\frac{3}{7}$ ,  $\cos \theta < 0$  and  $\tan \theta > 0$ , then find the value of:

a  $\cos \theta$

b  $\tan \theta$

c  $\theta$  to the nearest degree

12 Simplify each expression.

a  $\frac{\cot x \sec x}{\tan x \cos x}$

b  $\frac{\tan^2 x}{\sec^2 x}$

13 a Prove that  $(\sin x - \cos x)^2 = \sec^2 x - \tan^2 x - 2 \sin x \cos x$ .

b Using what you proved in part a or otherwise, prove that  $\sec^2 x - \tan^2 x - \operatorname{cosec}^2 x + \cot^2 x = 0$ .

14 Prove that  $\frac{1 + \sin x}{1 - \sin x} = (\sec x + \tan x)^2$ .

15 a In which quadrant does the angle  $\frac{16\pi}{3}$  lie?

b Hence find the exact values of  $\sin\left(\frac{16\pi}{3}\right)$  and  $\cos\left(\frac{16\pi}{3}\right)$ .

16 Solve  $3 \tan x - 1 = 0$  in the domain  $[0, \pi]$ , correct to 3 decimal places.

17 Sketch each trigonometric function in the domain  $[0, 2\pi]$ .

a  $f(x) = 3\cos\left(x - \frac{\pi}{2}\right)$

b  $g(x) = \tan x - 1$

c  $y = \cos(2x)$

d  $f(x) = 1 + \sin 3x$

**18** Find the domain and range of each function.

**a**  $y = -2 \cos x$

**b**  $y = 5 \sin (-3x)$

**19** Find the amplitude and period of  $y = -2 \sin 3x$ .

**20** Solve the equation  $\sin^2 \theta - 4 \sin \theta + 3 = 0$  for the domain  $[0, 2\pi]$ .

**21** Find the period, amplitude, centre and phase of  $y = \sin\left(2x - \frac{\pi}{4}\right) - 3$ .

### End of Part 1 Preparation Activity

