Name: _____

Year 11 2021 Mathematics Extension 1
Assessment Task 2

Assignment with Validation Task

Task number: 2	**Weighting: 30%**	**Due Date:** Wednesday 16/6/21

Outcomes assessed:

ME11-1	Uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses
ME11-2	Manipulates algebraic expressions and graphical functions to solve problems
ME11-6	Uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts
ME11-7	Communicates making comprehensive use of mathematical language, notation, diagrams and graphs

Nature and description of the task:

As a result of completing this Assignment, students should be familiar with all content related to the following topics:

- Polynomials and Inverse Functions ie. Chapter 6 of the Extension 1 Grove Book.

On the 16th June, 2021 you will receive a selection of similar questions to the Preparation Activity below to complete in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive in the in-class Validation task. NOTE: You will not have to hand in the answers to the questions in this Preparation Activity AND you will not have access to the Preparation Activity during the Validation Task.

Non-Completion of Task:

If you know you are going to be away on the day of the Validation Task and are unable to complete it on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

Part 1 Preparation Activity – investigate/attempt each of the following questions in preparation for the in-class Validation Task.

1 When dividing a polynomial $P(x)$ by $A(x)$, we can write $P(x) = A(x)\,Q(x) + R(x)$, where $Q(x)$ is the quotient and $R(x)$ is the remainder.
Which one of the following statements is true about the degree of $R(x)$?

A It is equal to 0.

B It is less than the degree of $A(x)$.

C It is equal to the degree of $A(x)$.

D It is less than the degree of $Q(x)$.

2 Find the values of m and n if $mx^3 + 7x + n$ has a remainder of 2 if divided by $(x - 1)$ and a reminder of 5 if divided by $(x + 1)$.

A $m = 5, n = 1$

B $m = -\frac{11}{2}, n = \frac{7}{2}$

C $m = -\frac{17}{2}, n = \frac{7}{2}$

D $m = \frac{17}{2}, n = -\frac{7}{2}$

3 Find the values of a, b, c, and d if $3x^3 - 7x^2 + bx + d \equiv ax^3 + cx^2 + 2x - 4$.

A $a = -3, b = -2, c = 7, d = 4$

B $a = 3, b = 2, c = 7, d = 4$

C $a = -3, b = 2, c = -7, d = 4$

D $a = 3, b = 2, c = -7, d = -4$

4 Which polynomial has a leading term of $3x^4$ and zeros of -3, 0, 1 and 2?

A $3x^4 - 21x^2 + 18x$

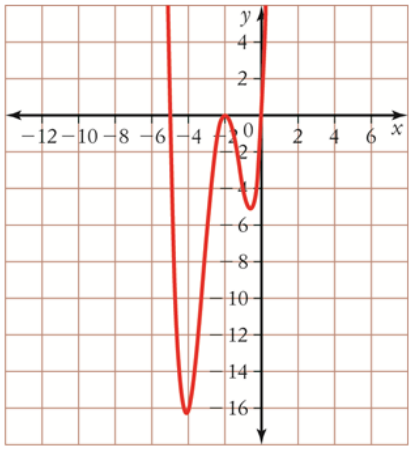
B $3x^4 - 6x^3 - 21x^2 + 18x$

C $3x^4 - 21x^2 + 18x - 6$

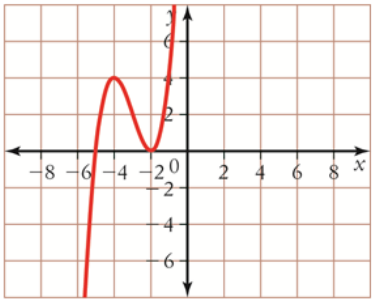
D $3x^4 - 21x^2 + 18x + 6$

5 Which of these graphs shows a monic polynomial with degree 4, a double zero at $x = -2$ and zeros at $x = 0$ and $x = -5$?

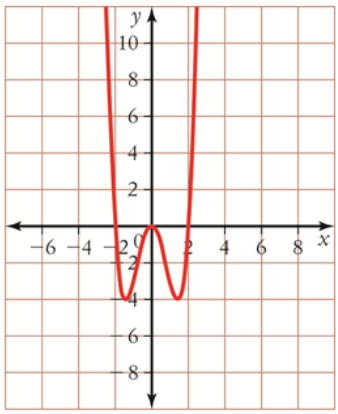
A



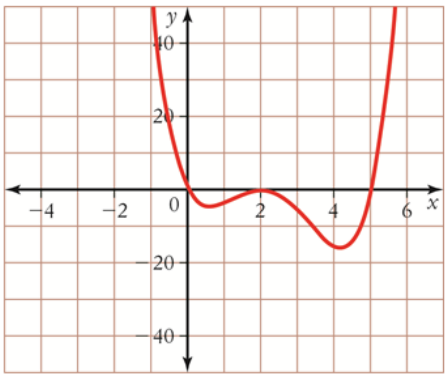
B



C



D



6 Find the inverse relation of $y = x^2 + 2$.

A $x = y^2 - 2$

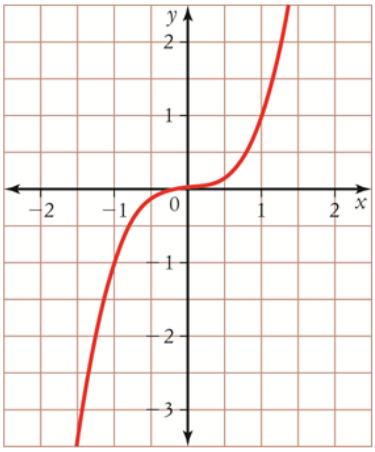
B $y = \sqrt{x - 2}$

C $y = \pm\sqrt{x - 2}$

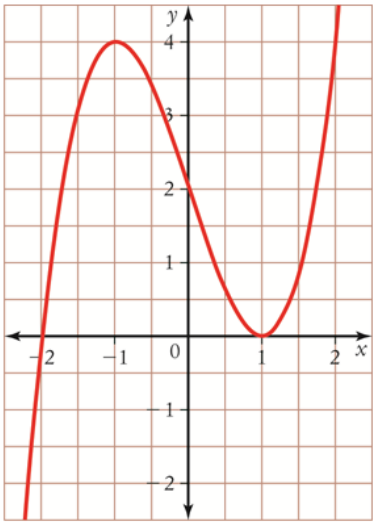
D $y = \pm\sqrt{x} - 2$

7 Which of these graphs shows a function that has an inverse function?

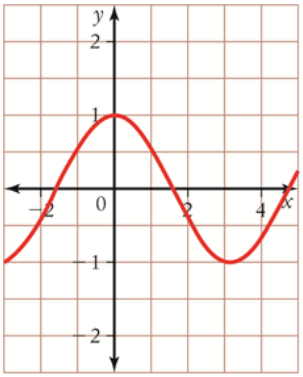
A



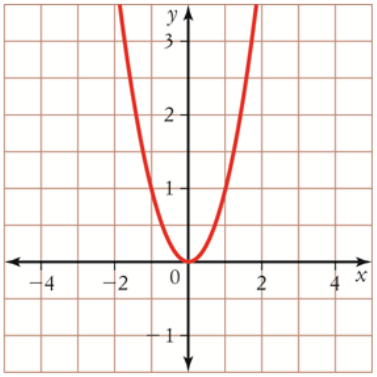
B



C



D



8 Which of these functions has an inverse function?

A $y = x^2 + 1$

B $y = (x + 1)^2$

C $y = x + 1$

D $y = \pm\sqrt{x+1}$

9 If $f(x) = x^2 + 2x + 3$, find its inverse function over the restricted domain $[-1, \infty)$.

A $f^{-1}(x) = -1 + \sqrt{x-2}$

B $f^{-1}(x) = -1 + \sqrt{x-4}$

C $f^{-1}(x) = -1 - \sqrt{x-2}$

D $f^{-1}(x) = 1 + \sqrt{x-2}$

10 Find the inverse function of $y = \dfrac{2}{x-1}$.

A $y = \dfrac{x-1}{2}$

B $y = \dfrac{x}{2} - 1$

C $y = \dfrac{2}{x} + 1$

D $y = \dfrac{2}{x} - 1$

11 Divide each polynomial $P(x)$ by $A(x) = x^2 - 2x$ and write $P(x)$ in the form $P(x) = A(x)Q(x) + R(x)$.

a $P(x) = 4x^4 - 3x^3 - 2x$

b $P(x) = 3x^3 - 7x + 2$

12 Prove that $(x - 5)$ is a factor of $P(x) = x^3 - 6x^2 + 3x + 10$, then find all the other factors and write $P(x)$ as product of its factors.

13 Find the remainder when $(2x^3 - 7x^2 + 2x - 1)$ is divided by $(x - 2)$.

14 Write $x^3 - 4x^2 - 11x + 30$ as a product of its factors.

15 $P(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$.

a What is the maximum number of roots possible for $P(x)$?

b Prove that $(x + 3)$ is a factor of $P(x)$.

c Find all the roots of $P(x) = 0$.

16 If α and β are roots of $x^2 - 2x - 5 = 0$, find:

a $\alpha + \beta$ and $\alpha\beta$

b $\alpha^2 + \beta^2$

c $\alpha^3 + \beta^3$

17 Find the roots of $x^3 - 5x^2 + 3x + 9 = 0$ given that 2 of the roots are equal.

18 Find the inverse of each relation.

a (1, –2), (2, –5), (3, –7), (6, 4)

b (–5, 2), (2, –3), (–3, 7), (7, –5)

19 Find the inverse function of each function and its domain and range.

a $h(x)=\sqrt[3]{x+1}+3$

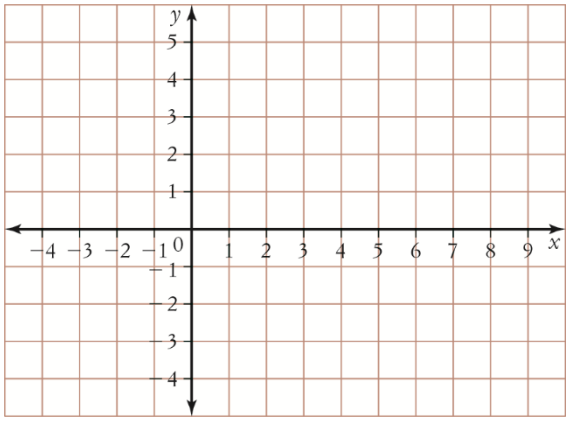
b $g(x) = 2x^3 + 3$

c $f(x)=\dfrac{1}{x+1}-2$

20 Graph $f(x)=\dfrac{1}{x-1}$ and $f^{-1}(x)$ on the same axes.

21 $f(x) = (x - 2)^2 - 2$

a Sketch the parabola $y = f(x)$. Clearly show all significant points.



b State the possible domain and range of $y = f^{-1}(x)$ and hence find all possible $y = f^{-1}(x)$.

c On the same graph as $y = f(x)$ in part **a**, sketch the graph of $y = f^{-1}(x)$ that is monotonically increasing, showing all important points. Label each graph clearly.

d Find the coordinates of the intersection points of $y = f(x)$ and $y = f^{-1}(x)$.

End of Part 1 Preparation Activity