**Name: _____**

# Year 12 2021 Mathematics Advanced
# Assessment Task 3

## Assignment with Validation Task

| **Task number:** 3 | **Weighting:** 25% | **Due Date:** Wednesday 19/5/21 |
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**Outcomes assessed:**

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|---|---|
| MA 12-1 | uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts |
| MA 12-3 | applies calculus techniques to model and solve problems |
| MA 12-6 | applies appropriate differentiation methods to solve problems |
| MA 12-7 | applies the concepts and techniques of indefinite and definite integrals in the solution of problems |
| MA12-10 | constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context |

**Nature and description of the task:**

As a result of completing this Assignment, students should be familiar with all content related to the following topics:

- Further Differentiation (Chapter 4 of the Advanced Grove Book and Chapter 5 of the Extension 1 Grove Book).

  NOTE: Further practice of Ex 4:08 Advanced Grove Book can be found in Ex 6:04 to Ex 6:08 inclusive. Similarly, further practice of Ex 5:08 Extension 1 Grove Book can be found in Ex 7:04 to Ex 7:08 inclusive. These types of questions may be tested but only the indefinite integrals.
- Geometrical Applications of Differentiation (Chapter 5 of the Advanced Grove Book and Chapter 6 of the Extension 1 Grove Book).

On the 19th May, 2021 you will receive a selection of similar questions to the Preparation Activity below to complete in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive in the in-class Validation task. NOTE: You will not have to hand in the answers to the questions in this Preparation Activity AND you will not have access to the Preparation Activity during the Validation Task.

**Non-Completion of Task:**
If you know you are going to be away on the day of the Validation Task and are unable to complete it on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

## Part 1 Preparation Activity – investigate/attempt each of the following questions in preparation for the in-class Validation Task.

### Further Differentiation:

**1** Find the derivative of each function.

**a** $f(x) = x^2 - 2x$

**b** $y = \frac{1}{x}$

**c** $y = \frac{x}{x^2 - 1}$

**d** $g(x) = (x^2 - x)^2$

**e** $h(x) = 4^x$

**f** $f(x) = 2xe^{x^2}$

**g** $f(x) = \log_e x^3$

**2** Find the anti-derivative of each function.

**a** $f(x) = 4x^3 - 3x^2 + x$

**b** $\frac{dy}{dx} = (5x - 7)^3$

**c** $g(x) = \tan^2 x \sec^2 x$

**d** $h(x) = \frac{4x - 1}{2x^2 - x}$

**e** $\frac{du}{dx} = xe^{x^2}$

**f** $y' = 2x^3 - 2x \sin x^2 \cos x^2$

**3** Find the equation of the normal to the graph for the function $y = x^3 + 2 \ln x^2$ when $x = -1$.

**4** A hyperbola has equation $y = \frac{2}{x}$. Find the equation of the tangent to the hyperbola at $x = 2$, in general form.

**5** Find the exact gradient of the tangent to the function $f(x) = e^{\ln x_2}$ at $x = \frac{1}{2}$.

**6** Find the equation of the function $f(x)$ if $f'(x) = xe^{x^2}$ and the point $P\left(0, \frac{5}{2}\right)$ lies on the graph of the function.

**7** Find $y = f(x)$ if $f''(x) = 3x^2 - 6x + 2$, $f'(2) = -3$ and $f(-1) = 2$.

**8** Find the derivative of $y = \sin 2x°$.

**9** The rate of change of volume $V$ with respect to time ($t$) is $\frac{dV}{dt} = (2t - 1)^3$.

If $V = 3$ when $t = 0$, find the volume when $t = 3$.

**10** Find the second derivative of $v = \frac{x}{x^2 - 5}$.

**11** Find each integral.

**a** $\int \sqrt{x} + \frac{2}{x^2} + e^{3-x} dx$

**b** $\int 3x \sec^2(3x^2) dx$

**c** $\int -\sin x \, dx$

**12** A particle has velocity $v = \frac{t}{t^2 + 1}$ m s$^{-1}$. Find its displacement after 4 s if its initial displacement is 3 m.

**13** A curve has its rate of change given by $\frac{dy}{dx} = \sin(3x)$ and passes through the point $\left(\frac{\pi}{2}, \pi\right)$.

Find the equation of the curve.

## Geometrical Applications of Differentiation:

**1** For what values of $x$ is the graph of $y = 3x^2 - 4x + 1$ increasing?

**2** The curve $f(x) = x^3 + ax^2 + bx - 4$ has stationary points at $x = 3$ and $x = 5$. Find the values of $a$ and $b$.

**3** Draw a sketch of a function $y = f(x)$ where $\frac{dy}{dx} > 0$ for $x < 5$, $\frac{dy}{dx} = 0$ when $x = 5$ and $\frac{dy}{dx} < 0$ for $x > 5$.

**4** Consider the function $f(x) = 2x^3 + 6x^2 - 1$ in the domain $[-3, 1]$.

**a** State the $y$-intercept.

**b** Locate any stationary points and determine their nature.

**c** Determine the point of inflection.

**d** Hence sketch the graph of $f(x) = 2x^3 + 6x^2 - 1$ in the domain $[-3, 1]$.

**e** What is the global maximum value of the function in this domain?

**5** Consider the graph of the function $y = 2x^3 - 6x^2 + 6x + 1$.

**a** Show that the graph has only one stationary point, find its coordinates and determine its nature.

**b** State the values of $x$ for which the curve is concave up.

**c** State the values of $x$ for which the curve is increasing.

**6** Consider the function $y = x\sqrt{4 - x^2}$.

**a** What is the domain of this function?

**b** Find the stationary points for this function and determine their nature.

**c** Sketch the graph of the function.

**7** The 3 dimensions (length, breadth and height) of a box with a square base of side $x$ all add up to 40 cm. Show that the volume of this box is given by $V = 40x^2 - 2x^3$ and hence find the maximum volume of this box.

**8** A scenic helicopter flight can take $x$ passengers per week for a charge of $(200 – 0.3x)$ dollars per person. The number of flights is not relevant. The cost of operation each week is $\$(3000 + 50x)$ dollars when $x$ passengers
are taken.

**a** Show that the profit, in dollars, per week is given by $P = 150x – 0.3x^2 – 3000$.

**b** How many passengers should be taken, per week, to maximise the profit?

**c** Hence, given this number of passengers, what is the charge, per passenger, when this maximum profit occurs?

## End of Part 1 Preparation Activity