Name: ____________________

Year 12 Mathematics Extension 1 HSC Assessment Task 3

Trigonometric Functions, Derivative of Inverse Functions & Derivative of Inverse Trigonometric Functions		
Task number: 3	**Weighting: 25%**	**Due Date: 2/6/21**

Outcomes assessed:

ME 12-1 applies techniques involving proof or calculus to model and solve problems

ME 12-3 applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

ME 12-4 uses calculus in the solution of applied problems, including differential equations

Nature and description of the task:

As a result of completing this Assignment, students should be familiar with trigonometric identities, auxiliary angle results, further trigonometric equations, general solutions and their graphs. They should be familiar with the derivative of inverse functions and the derivative of inverse trigonometric functions. They should be familiar with the mathematical language, notation and graphs associated with these functions.

On the 2nd of June, 2021 you will receive a similar selection of the following questions from the Preparation Activity below to complete in 50 minutes in an in-class Validation Task. You are expected to investigate/attempt each of these questions before the in-class Validation Task. The final mark for this assessment will be the mark you receive on the in-class Validation task. NOTE: You will NOT have access to the Preparation Activity during the Validation Task. You will NOT be given any answers to the Preparation Activity. You will be given a HSC formula sheet.

Non-Completion of Task:

If you know you are going to be away on the day the Assessment Task is due and are unable to hand in Assignment on the due day, then you must have supportive documentation. *Zero marks will apply if the Assessment Task is submitted/completed late, unless an Illness/ Misadventure or Application for Extension form has been submitted.*

PREPARATION ACTIVITY

PART A

1 Find the derivative of:

a $y = \cos^{-1} 2x$

b $y = \tan^{-1} \frac{x}{2}$

c $y = \sin^{-1}(1 + x)$

d $y = x^3 \tan^{-1} x^2$

e $y = \tan^{-1}\left(1 - \frac{3}{4}x\right)$

f $y = \cos^{-1} \frac{1}{x}$

2.

Find the gradient of the tangent to each curve at the point indicated:

a $y = 2\tan^{-1} x$, at $x = 0$

b $y = \sqrt{3}\sin^{-1} x$, at $x = \frac{1}{2}$

c $y = \tan^{-1} 2x$, at $x = -\frac{1}{2}$

d $y = \cos^{-1} \frac{x}{2}$, at $x = \sqrt{3}$

3.

Find, in the form $y = mx + b$, the equation of the tangent and the normal to each curve at the point indicated:

a $y = 2\cos^{-1} 3x$, at $x = 0$

b $y = \sin^{-1} \frac{x}{2}$, at $x = \sqrt{2}$

4.

Find the derivative of each function in simplest form:

a $x\cos^{-1} x - \sqrt{1 - x^2}$

b $\sin^{-1} e^{3x}$

c $\sin^{-1} \frac{1}{4}(2x - 3)$

d $\tan^{-1} \frac{1}{1 - x}$

e $\sin^{-1} e^{x}$

f $\log_e \sqrt{\sin^{-1} x}$

g $\sin^{-1} \sqrt{\log_e x}$

h $\sqrt{x}\sin^{-1} \sqrt{1 - x}$

i $\tan^{-1} \frac{x + 2}{1 - 2x}$

5. Differentiate the inverse function of:

a) $y = x^4 + 3$

b) $y = (x+4)^3$

c) $y = \sqrt{x}$

6. Show that $\frac{dy}{dx} \times \frac{dx}{dy} = 1$ given:

a) $y = 3x + 4$

b) $y = x^3 - 1$

c) $y = e^x$

7. Find the gradient of the (i) Tangent (ii) Normal to the inverse function at $(\frac{1}{9}, 2)$ given

$f(x) = \frac{1}{x^3+1}$

8. Find $\frac{dx}{dy}$ of the inverse function $f^{-1}(x)$ of each function in terms of y.

a) $f(x) = x^2 e^x$ b) $f(x) = \frac{4x-1}{2x+3}$ c) $f(x) = 3x\sin 2x$

PART B

1 Solve, for $0 \le x \le 2\pi$:

a $\sin x - \sin 2x = 0$ b $\cos 2x - \cos x = 0$

c $\sin 2x + \sqrt{3}\cos x = 0$ d $\sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2}\cos\left(x - \frac{\pi}{6}\right)$

2 a Express $\sin x + \cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

b Hence solve $\sin x + \cos x = 1$, for $0 \le x \le 2\pi$.

3 a Express $\sqrt{2}\cos x - \sqrt{2}\sin x$ in the form $A\cos(x + \alpha)$, where $A > 0$ and α is acute.

b Hence solve $\sqrt{2}\cos x - \sqrt{2}\sin x = 1$, for $0 \le x \le 2\pi$.

4 a Express $\sqrt{3}\sin x + 3\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and α is acute.

b Hence solve $\sqrt{3}\sin x + 3\cos x = 3$, for $0° \le x \le 360°$, writing the solution in degrees.

5 a Express $-2\sin x + \sqrt{5}\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and α is acute. Write α correct to the nearest minute.

b Hence solve $-2\sin x + \sqrt{5}\cos x = 2$, for $0° \le x \le 360°$, writing the solution correct to the nearest minute where necessary.

6 Use a suitable t-formula to solve $\cot^2 x = \tan\frac{x}{2}$, for $0 \le x \le 2\pi$.

7 Consider the equation $8 \sin x + 6 \cos x = 8$.

a Show that the equation can be written as $14t^2 - 16t + 2 = 0$, where $t = \tan \frac{x}{2}$.

b Hence show that the equation has solutions $x = 2 \tan^{-1} \frac{1}{7}$ and $x = \frac{\pi}{2}$.

8 Use t-formulae to solve $4 \cos x - 6 \sin x = 3$, for $0 \leq x \leq 2\pi$.

Write the solutions correct to two decimal places.

9 a Use compound and double-angle formulae to prove that $\sin 3x = 3 \sin x - 4 \sin^3 x$.

b Hence solve $\sin 3x + \sin 2x + \sin x = 0$, for $0 \leq x \leq 2\pi$.

10. A spring moves so that its displacement is given by $x = 7\cos \frac{\pi t}{3} + 4$ cm over t secs.

a) Find the maximum displacement and the times at which this occurs.

b) Find the minimum displacement and the times at which this occurs.

c) When is the spring at the centre of its motion?

d) Solve $7\cos \frac{\pi t}{3} + 4 = 7.5$

11. Find the general solutions of

a) $\cos x = \frac{\sqrt{3}}{2}$

b) $\tan x = 1$

c) $\sin x = 0$

END OF PREPARATION ACTIVITY